Mathematics vs. 'New Math'—how it should be taught

by Dr. Steven Bardwell

The multitudes of American parents who have felt frustration and rage at what passes for mathematics in today's schools, the parents who have, in the end, resigned themselves to the fact that Johnny can't add, unconsciously adhere to a long, continuous line of mathematical thought—stretching from the mathematicians of Plato's Academy, Archimedes, through Nicholas of Cusa and Leibniz to the great 19th-century school of German and French mathematicians. This tradition is outstanding for two reasons: first, its members are responsible for *every essential* mathematical discovery in the last 2,000 years, and second, it has been pitted, since its inception, against a contrary tradition in mathematical thinking; today's parents are the frontline of that fight.

The New Math is not really new, any more than the inspiration for its method is new. Lord Bertrand Russell and Swiss "child psychologist" Jean Piaget, are the modern progenitors of the development of the New Math's ideas. Both are quite explicit that their aim is to establish a non-Platonic mathematics, based on the methods of Aristotle; both make unmistakably clear that the fundamental issue is one concerning *how* men think:

> The 'rational nature' of man is only a derivative. The subject and object of knowledge are separate. ... On this point as on many others' Aristotelean physics marks a return to ordinary thought rather than a continuation of the aspirations of Platonist mathematics.
>
> Jean Piaget: *Mathematical Epistemology and Psychology*

On the other side, perhaps the clearest statement of the Platonic view is given in a paper by the founder of the *real* theory of sets, Georg Cantor:

> We can speak of the reality or the existence of the whole numbers, both the finite and the infinite ones in two senses; however, these are the same two ways, to be sure in which any concepts or ideas can be considered. On the one hand we may regard the whole numbers as real insofar as they take up a very definite place in our mind on the basis of definitions, become clearly differentiated from all the other components of our thinking, stand in definite relations to them and thus modify the substance of mind in a definite way. Let me call this type of reality of numbers their intrasubjective or immanent reality. Then again we can ascribe reality to numbers insofar as they must be regarded as an expression or image of occurrences and relationships in the external world confronting the intellect. This second type of reality I call the transsubjective or transient reality of the whole numbers....
>
> There is no doubt in my mind that these two types of reality will always be found together, in the sense that a concept to be regarded as existent in the first respect will always in certain, even in infinitely many ways, possess a transient reality as well....
>
> This coherence of the two realities has its true foundation in the *unity* of the *all, to which we ourselves belong as well.*

This view of mathematics and science is what the New Math is designed to destroy. The Platonists have maintained that mathematics is an empirical science whose subject (like that of any science) is what Plato called the "hypothesis of the higher hypothesis" and Cantor called the "Principle of Generation," both descriptions of the self-developing evolution of the Universe. The Aristotelian opposition has counterposed the view that mathematics (along with the other sciences) is a logical structure, lacking any *essential* connection to reality, and *merely* a product of the human mind, a mind which in their view has itself no essential connection to reality. (This psychology is obviously self-validating, as the insanity of many of the most illustrious of the latest generation of mathematicians testifies).

The fight between these two views in the 20th century has taken place over the basic concepts of arithmetic numbers and arithmetic operations. The biggest guns of the Aristotelian faction have, in fact, been aimed at overturning the explicitly Platonic significance of the concept of number developed, as both sides recognize, by the discoverer of set theory, Georg Cantor.

Bertrand Russell spent ten years of his life producing a three volume book, *Principia Mathematica*, which he hoped would show that mathematics could, *through the use of set theory,* be reduced to logic. He failed, but his book became the model for three generations of formal logical mathematics to be used against Platonic methods in mathematics. On the pedagogical side, Jean Piaget took Russell's work and developed a theory of number and the concept of number which he claims purifies Cantor of his Platonic excesses!

The new math is the fruition of the Piaget-Russell attack on Platonic mathematics. Its incoherence, self-evident sterility, and destructive effect on children's minds are not accidental—this is the essence of the Aristotelian theory of mind.

Two examples

There have been many attacks on the New Math, but its epistemological significance remains largely unknown. The destructiveness of the New Math is clear from two examples taken from its curriculum which have escaped the notice of critiques of New Math from conventional or practical standpoints; I want to concentrate on these here. The first is the concept of an "algorithm" which is used as the basis for teaching arithmetic operations, and, second, the New Math concept of the structure of the number system.

1. Algorithms and arithmetic

An algorithm is a set of rules, usually recursive, for performing some task and for testing for the completion of the task. The concept of an algorithm was a product of the development of machines which had to be "programmed" with instructions for the actions required of the machine. The punched cards that controlled early spinning and knitting machines are classic examples of an algorithm—move needle A to position 1, needle B to position 2, move the red thread over needle 1, etc. Obviously, an algorithm is a powerful tool if certain conditions are satisfied:

1) The problem to be solved or task to be performed is completely posed beforehand;

2) The problem can be solved in a finite number of steps;

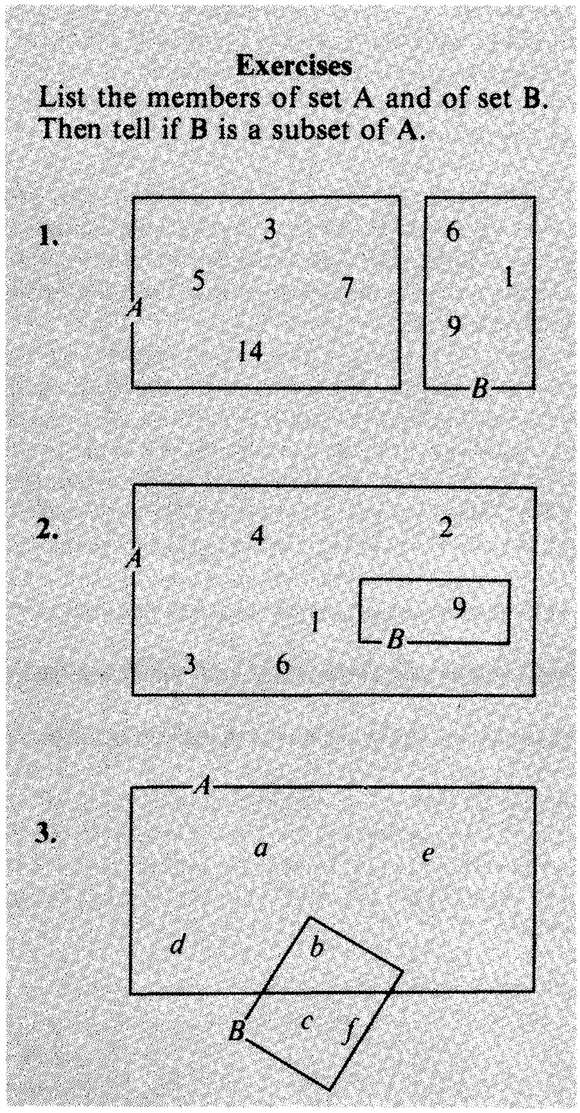


FIGURE 1

Set theory vs "set theory"

The new math is well known for its love of set theory, as exemplified by the above diagram from a first grade workbook illustrating the idea of number as a property of sets of arbitrary objects. However, the set theory taught in the new math is diametrically opposed to the concept as developed by its inventor, Georg Cantor. As Cantor makes clear over and over again, a set is *not* an arbitrary collection; it is defined by the "rule" which determines membership in a set. There are real sets and collections which are not sets. Cantor put into mathematical form, with his definition of sets, the essentially Platonic idea of a universal—a "set" is a higher-order concept, not a simple aggregation of objects. The new math, based on Russell's bowdlerization of set theory, turns the whole concept into a nominalist game. As the above picture shows, any collection can be a set even if the "rule" for membership is a totally arbitrary one.

3) The quality of solution does not depend on factors known only after the algorithm is begun (for example, singularities are excluded);

4) The rules for performing the algorithm are fixed or drawn from a fixed group.

These assumptions are fine for a machine or a computer, but they are all violated by the simplest task required of human mentation! No algorithm could be written for something as simple as getting out of bed (or getting your kids to school) in the morning.

In spite of this obvious fact, the algorithm has been taken as a prototype of mathematical thinking by the Aristotelians and incorporated in the New Math as the way of teaching arithmetic operations like addition and subtraction. From a psychological and pedagogical standpoint this is absurd. Since people are not machines they perform tasks differently and they learn them differently. In the same way, this method is absurd mathematically; arithmetic operations are only *formally* reducible to algorithmic techniques. They are actually synthetic concepts, higher order concepts, and, when reduced to their algorithmic counterpart, cease to be mathematics.

Long-division, long the terror of elementary children students, provided fertile ground for the New Math's algorithmic theory of arithmetic. Presented with the problem of dividing 90 by 8, the New Mathematician will tell us the following (of course, he probably won't actually *do* the division this way—but this is what he says to the kids):

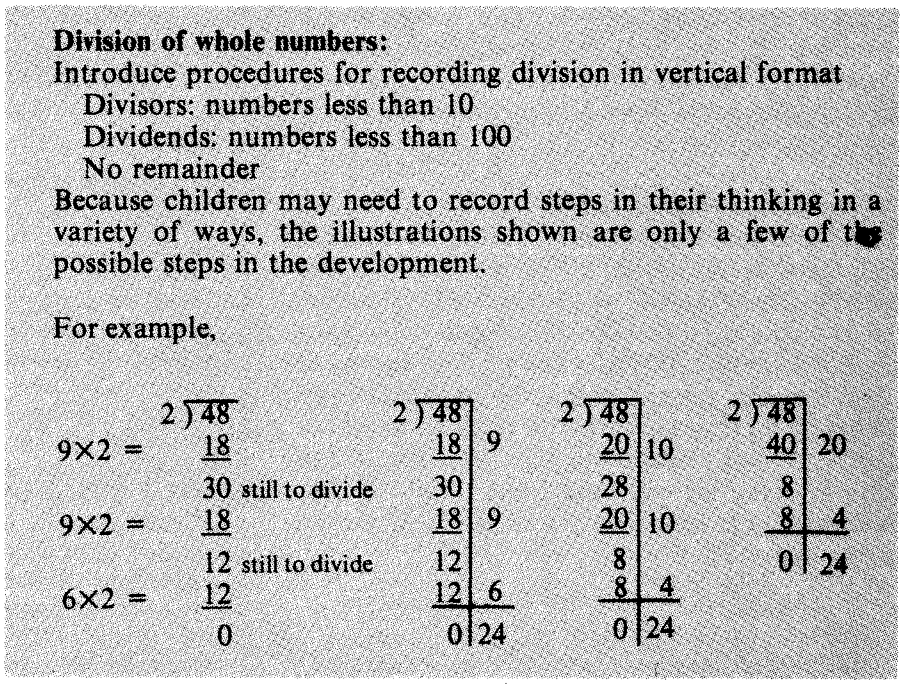
STEP 1: Is 8 larger than 90? If yes, then quotient is 0; otherwise go to STEP 2.
STEP 2: Subtract 8 from dividend. Add 1 to quotient.
STEP 3: If 8 is larger than dividend then end;

The algorithm which he proposes counts the number of times that the divisor (8) can be subtracted from the dividend (90)—this number of times is the quotient (11). This method is used, in actuality, only by the crudest of mechanical calculators—even computers have better ways of dividing!

Is this algorithm even division? Let's try it on the problem of 4 divided by 12—the answer, according to one student is −8. Certainly not. From a mathmatical standpoint, division is qualitatively different from subtraction—it is not compounded subtraction, unless, of course, you are a mechanical calculator. Subtraction of whole numbers, no matter how many times it is performed, always produces whole numbers; but division, takes whole numbers and produces a new kind of number—a rational number, or fraction. One can never get fractions from subtraction of whole numbers.

This reduction of division to an algorithm involving repeated subtraction is not merely a mathematical travesty. The subject of mathematics, as all great mathematicians have known, is not numbers and their manipulation; it is the human mind

FIGURE 2
Division as repeated subtraction
The algorithmic approach to arithmetic shows its impracticality and its inaccuracy in this figure taken from a fourth grade new math text. The diagram is an attempt to show how long division can be done by counting the number of subtractions of the divisor from the dividend.


Division of whole numbers:
Introduce procedures for recording division in vertical format
Divisors: numbers less than 10
Dividends: numbers less than 100
No remainder
Because children may need to record steps in their thinking in a variety of ways, the illustrations shown are only a few of the possible steps in the development.

For example,

as a mirror of the Universe. Mathematics, as a product of the human mind, both reflects and modifies the structure and evolution of the Universe. Cantor says that this connection—the "unity of the all"—*is* mathematics. Since neither the human mind nor the Universe satisfies any of the four prerequisites for the applicability of an algorithm, to teach algorithmic thinking as if it were mathematics is to systematically distort both reality and human mentation. No wonder children hate the New Math—to understand it, they must deny the fundamental characteristic of their ability to think!

Let there be no mistake; the Aristotelian faction of mathematics agrees about the implications of algorithmic thinking. They only disagree about the inapplicability of algorithmic methods to the mind and the Universe. Their premise is that the laws governing both human thought and the Universe *are* fixed. Of course, they say, algorithms work precisely *because* human beings and the Universe are machine-like.

2. The Structure of the Number System

The problem of long division raises a more fundamental problem in arithmetic; the New Mathematician's reply to my objection that his algorithm for long division could not generate fractions (because subtraction of whole numbers can only generate whole numbers) would be the following: I can provide you with an algorithm that is too simple, but just because subtraction doesn't give you fractions, doesn't mean that there is no algorithm for doing so.

The real argument here is not over an algorithm for long division, but rather, over the significance of these new numbers generated by division. Any qualitative significance of division comes from its ability to generate these new numbers (fractions). The Platonic approach to mathematics has maintained, as Cantor and Dedekind were the first to show, that fractions (rational numbers) are a qualitatively different kind of number than whole numbers. In addition, Cantor showed that the number system is, in fact, a nested hierarchy of different kinds of numbers, each of which is generated from the preceding by inherently nonalgorithmic processes like limits of infinite series. To get irrational numbers from rational ones, for example, requires a complicated geometrical argument that demands new mathematical rules for new numbers.

As Cantor points out, the significance of this hierarchical structure of the number system far transcends its mathematical applications. It is parallel to—a model of—the similar nested, hierarchical structure of the physical Universe. Cantor showed, even more, that the fundamental feature of this hierarchy was not its structure at any one instant, but rather what he called the Principle of Generation which creates a new level of hierarchy out of its predecessor. This transition from one level to the next (like from the whole numbers to the rational numbers) is lawful but there is nothing in the lower level that *determines* beforehand its successor. The Principle of Generation in mathematics has been called "negentropy" in physics—but they are the same.

By his algorithms, bastardized set-theory, and the like, the New Mathematician denies the qualitative structure of the number system. The crux of the Aristotelian approach is that the Platonic hierarchy does not exist. Russell's book was an attempt to prove the qualitative homogeneity of mathematics—to prove that it was *in toto* reducible to a fixed set of logical axioms. If he had been successful, it would have been possible to build a computer which could prove every existing theorem in mathematics and every theorem ever provable! He was not successful, but not because of any shortcoming of his attempt; it is just that he and his New Math disciples are wrong about the nature of the human mind and physical universe.

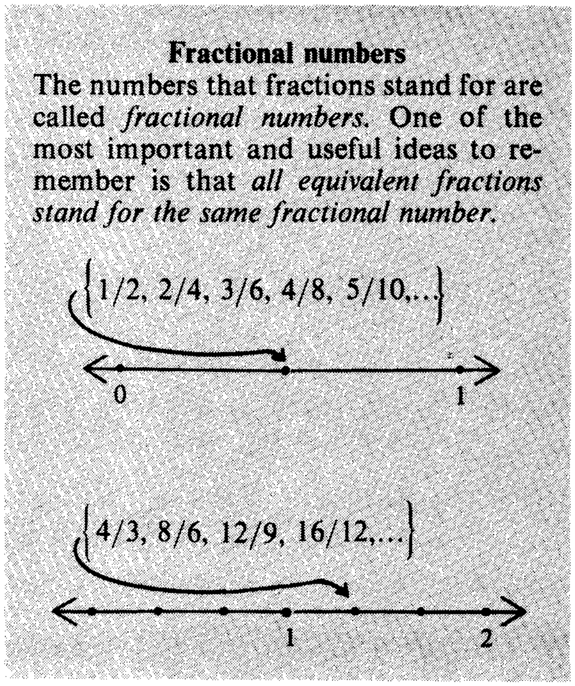


FIGURE 3
The number line

Contrary to the implication of the new math, the number line (the continuum) has a subtle and important structure. All numbers on the number line are not the same, and, as Cantor stressed, the generation of one kind of number from a simpler one is a prototype not only for all mathematical reasoning but also for the evolution of the universe.