No limits to growth: Cantor's concept of infinity in economic science

by Lyndon H. LaRouche, Jr.

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In 1854, at the conclusion of his habilitation dissertation, Bernhard Riemann, a protégé of Carl Friedrich Gauss and of Lejeune Dirichlet, made the following statement, which, together with the dissertation as a whole, represented, for those who were willing to understand, an absolute revolution in mathematics and mathematical physics. The statement is simple. Having made his remarks on mathematics, he said, "This path leads out into the domain of another science, into the realm of physics, into which the nature of this present occasion"—referring to the habilitation on mathematics—"does not permit us to enter."

This same observation was made, as I shall indicate, about 2,300 years earlier than that meeting in Göttingen (just over the hill, so to speak), by Plato. The particular work in Plato (and there are many which are relevant to this point), which is most relevant to the content of Cantor's work, is the famous "Parmenides" paradox, or the ontological paradox in "Parmenides," which is what Cantor is addressing.

Then, of course, about 30 years after that habilitation dissertation by Riemann, we had the *Grundlagen*[1] and some other writings and letters by Cantor, which also attacked the same problem, but from a completely different method. The method of Riemann was the method of geometry; and, although Cantor makes reference to geometry, his method is not that of Riemann, but rather of one of his teachers, Karl Weierstrass. So there's a difference in method between the two approaches, though they converge on the same conclusion.

Also notable is that, during the middle of the 1880s, as in the *Mitteilungen*[2] and in later writings, the statements of Cantor on the subject of Plato are *very significant* for a better understanding of both Cantor and Riemann. That is, Cantor emphasizes that the universe that exists, is the universe of the *Becoming,* not a fixed, empty space-time with objects floating around in it. And that is, he says, the *Transfinite,* by which he means, inclusively, what becomes known as the Aleph series. He identifies the *Good* of Plato as his notion of the *Absolute,* of God.

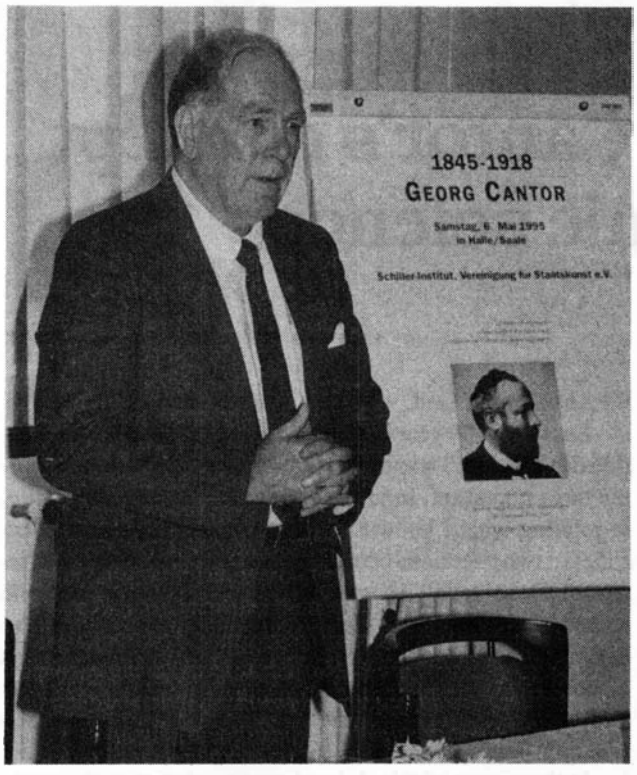
Less than 100 years after Riemann presented his habilitation dissertation, I was in the midst of a major work, probably the most important, in terms of practical results, in my life, in refuting what I knew to be the immoral and absurd doctrines of two twentieth-century gentlemen: one, Norbert Wiener, of the so-called information theory, and what goes with it; and the other, John Von Neumann, a man whom I've described often enough as virtually an *idiot savant,* a man whose head could juggle mathematical symbols at a great rate, and great numbers, but could *never* actually master a scientific concept.

In the course of this, I developed a solution to this problem, of how to refute these two gentlemen, based on the work I'd done earlier in refuting Kant, in defense of Leibniz. But the question was: Having made this discovery, how could it be made representable? How could we apply the discovery which I had elaborated in terms of economics, to *measure,* as we must, in making economic or related policy, or in measuring certain kinds of results?

In that context, I came across Cantor's work, especially the *Beiträge,*[3] which I spent about six months struggling through, before getting some comprehension of what the work was. Then, on the basis of having read Cantor, including the *Beiträge,* particularly the important notion of the *power series,* the power set, I returned to read Riemann again, and this time with proper understanding.

Riemann's discovery

To situate this matter, let me first indicate what Riemann's discovery is, why it is so fundamental, and why it leads to a notion of physical science which is directly contrary to that which is generally accepted still today, in the university classroom. The conception of mathematics and physical science as defined by Riemann's successful discovery as a young professor, is *still* not understood today, and *refused absolutely* in what is generally accepted as the notion of scientific method, both in mathematics as such and in physics in general, in the university classroom of today, and is also rejected, absolutely, by the *Brotgelehrten* ["bread scholars"—ed.] who are called today's economists, who know nothing, but who have much authority to speak about it at great length.



Lyndon LaRouche speaks in Halle, Germany, where Cantor spent much of his life.

Riemann's discovery is a simple one, which, in principle, was known before him. It was known first by Plato, in a formal, rigorous way, and then it was addressed by others, including Leibniz. But, as Riemann says, the problem of geometry had not been effectively attacked up to his time, from a formal standpoint. He had a discovery in this connection, and had spent from about March of 1853 through June of 1854 on a special research permission on the university campus at Göttingen, to do research into every possible source, to determine if there were any indications in previous writings in mathematics and physics, which might pertain to his insight about the problem of geometry. He said he found only a couple of references, and emphasized the notions of general curvature of curved surfaces by Gauss, as being the only method by which you could attack, practically, the problem of geometry as he understood it.

Let me state the problem in my own terms, because those are more relevant to what I shall treat from this standpoint later in my presentation.

In Euclidean geometry, or any similar geometry, we use two methods. One is the method of construction, the other is the method of so-called deductive or inductive proof. In that case, given a proposition, we submit the proposition to the principles of existing geometry. And we reject, as a proposition, at least as a theorem of that geometry, any proposition which is *inconsistent* with the existing body of so-called proven theorems.

Now, this implies, especially from a Socratic standpoint (and this, of course, is famously reflected in Euclid in part—not adequately, but in part), that what makes it possible to combine all theorems into a set of mutually consistent propositions, defines certain common or underlying assumptions in that geometry. These assumptions are called, classically, *axioms and postulates*. A *theorem-lattice* so defined is, in a formal sense, viewed as a collection of interdependent, that is, not inconsistent, axioms and postulates, none of which must be contradicted by any proposition which is accepted as a theorem. . . . To think about a geometry, we think not about a collection of theorems, we think of a theorem-lattice as a whole set of all possible propositions which might be consistent or not inconsistent with the underlying set of axioms and postulates.

So, in order to understand a geometry, instead of looking at the theorems one by one, we now look at the *common principle* which is referenced, by comparing that set of axioms and postulates with the set of axioms and postulates of any different kind of geometry. So we go up two higher steps in thinking from the level of theorems and propositions, first of all, to understand all possible theorems as a whole, as a kind of transfinite collection, in terms of thinking about the set of axioms and postulates. In order to understand axioms and postulates, to criticize them, we must make axioms and postulates, as a set, an *object of thought,* a subject of thought.

Thus we must think about *all possible* theorems and postulates, a still higher step, and look down, as it were, upon any particular set of axioms and postulates as merely one element of a large series.

These ideas of *hypothesis,* or rather, of axioms and postulates, have a very specific form in Plato. In a formal theorem-lattice, any given set of axioms and postulates is what Plato calls an *hypothesis*. Thus, all Euclidean geometry constitutes, really, *one hypothesis*. The introduction of non-Euclidean geometry in various ways, or corrections in Euclidean geometry—which become obvious partly with Nicolaus of Cusa, which develop in the work of Leonardo da Vinci, which appear prominently in Leibniz and so forth, and then emerge as the non-Euclidean geometries of the nineteenth century—is the standpoint from which we look today, as did Riemann, at Euclidean geometry, or similar geometries.

So therefore, we have to think about a *generality of geometries,* in terms of different sets of axioms and postulates, which sets of axioms and postulates are, shall we say, genetic in quality, so that you might say that a Euclidean geometry is a marsupial mammal, and a non-Euclidean geometry is a placental mammal, a higher form of life.

Riemann's work is one of the most important developments in non-Euclidean geometry, *the* most important, because it attacked explicitly something which Gauss in part knew, but in Gauss's letters to János and Farkas Bolyai, Gauss admits that he had *political fears* which prevented him from ever, in his lifetime, presenting his own discoveries in non-Euclidean geometry—*political fears* within the bounds

of official, institutionalized science itself.

Riemann was the first to openly challenge the assumptions of a formal geometry.

The fallacy of 'classroom mathematics'

What's the fallacy, looking from a higher standpoint? The fallacy is the idea that I call "the geometry of the naive imagination."

What is considered a naive geometry, is a commonplace geometry of any ignorant man in the street who says that in space there are three directions: forward/backward, up/down, and sideways, side to side; that in time there is only backwards and forwards. And thus we have the typical notion of space-time. In addition to that, this notion of space-time, in the naive imagination, is associated with infinite or unlimited extension, backward and forward. Up to 1963, we went forward, since 1964, we've gone backward, as in economy. So you can see how we go backward and forward in time.

It also was assumed that the extension of unlimited extension in space and time, is "infinitely divisible." There's a famous case of this in 1761, when a man who was a great mathematical talent but a personally immoral person, Leonhard Euler, wrote a series of papers, in his "Letters to a Princess," denouncing the *Monadology* of Leibniz. The argument he used was a very simple, crude, and immoral construction of infinite divisions. These are natural assumptions of a simple, Euclidean-style space-time.

This is the foundation of the mathematical theory, for example, of one of the most evil men of modern history, Paolo Sarpi of Venice, the man who did more to shape current history, perhaps, than any other single individual—at least, of all the bad ones—and his student, who was also a totally immoral person, but who followed totally his master's theory: Galileo Galilei. And also Thomas Hobbes, who was a student of Galileo, as well as a lover of Francis Bacon, who was also a Sarpi protégé; and then, of course, Descartes. Through the efforts of Sarpi's follower, another Venetian gentleman by the name of Antonio Conti, they took an obscure and rather eccentric, superstitious, Black Magic practitioner from Cambridge University, who happened to be an official of the London Royal Society, Isaac Newton, and they apotheosized him from the gutter of science to become the famous Newton, and used this image of Newton to destroy science throughout much of Europe up until the fall of France in 1814. After the destruction of the major resistance to superstition, the Ecole Polytechnique under Monge and Lazare Carnot, then the superstitious fellows, such as LaPlace and Cauchy, took over the Ecole Polytechnique, destroyed its curriculum, destroyed its pupils, and began to produce the forms of Black Magic which emanated from France around the world, called "political science," "sociology," "ethnology," "anthropology," and "modern psychology"—all of these pseudosciences.

This same tradition dominates the classroom. The appearance of Paolo Sarpi divided all of prominent European science into two currents. One current leads through people like Kepler, Desargues, Leibniz, and so forth, through people like Gauss and Riemann. The other current of science, the counter-current, is the current of Sarpi; his student Galileo; his student Thomas Hobbes, who's a mathematics student who developed sociology from Galilean mathematics; Newton; Euler, who, even though he's a clever mathematician, is a complete prostitute morally in science; and then we have Clausius, Helmholtz, and so forth, in the nineteenth century.

As Riemann says, to make an advance in mathematics, you must step outside of mathematics, into the realm of physics. If you enlarge that, as I do, you will say, "Yes, this is true; but let's go one step further, into the physical economy, the process of reproduction of the society, which must become more suitable to the individual made in the image of God. . . ."

The last great scientist who can be said to belong to the tradition of Leonardo and so forth, is probably Max Planck. And the terrible things that were done, almost a Ku Klux Klan lynch mob attack on Max Planck, during the period of the First World War in Germany, are an example of how science was essentially destroyed.

There are many people today who are, I would say, good scientists, in the sense of being good engineers, and occasionally you'll find some eccentric person who's actually a scientist, who will challenge the assumptions of the generally accepted classroom mathematics; but there are very few of them who, trying that, survive. Usually, when the scientific community finds somebody who violates that principle, they will either teach him mathematics until he goes insane, or they will destroy him by other means. You might say that insanity among scientists is mathematics continued by other means.

Now, what Riemann did in reference to physics, simply, was this, and the solution was obvious to me from the standpoint of economics. Let me take another train of thought on this, and bring the two together, to indicate this.

What the Renaissance achieved

As I've said often, until the fifteenth century in Europe, over 95% of all mankind lived in a condition, in greater or lesser degree, comparable to that of human cattle. They lived

close to the soil, usually in rural or other occupations of that sort, had very short life expectancy, high infant mortality, long, hard hours of work. They were ground down, and they died often and early; and they were treated like human cattle by the top layer of society, which was less than 5% of the total population of every society prior to the fifteenth century in Europe.

At the top of society, was a group of families who fancied themselves like the gods of Olympus in the pagan writings of Hesiod and others of that time, people who played with ordinary human beings, even with their lackeys, at whim: "Kill him." "Kill him," "Kill him," "Beat him," "Destroy him," "Destroy that people," "Destroy that village." The idea of *natural law* and the natural right of human beings, for these oligarchs, did not exist. They were like the gods of Olympus, as described by Aeschylos in the *Prometheus Trilogy,* at least the first part which has survived.

In the period around the Council of Florence, and with the development of modern France by Louis XI in 1461, through the 20-odd years of his reign, there came a new form of society, the modern nation-state, committed to scientific and technological and other progress, in the conditions of life of the people within it. The characteristic feature of this was education, as begun earlier by Augustinian groups, teaching groups, by the Brotherhood of the Common Life, and so forth, which reached out and picked up boys who came from, generally, poor families, including orphans, and pulled them into a secondary education of the type we would call today a *Classical humanist model of education.*

These young boys did not have textbooks, which was one of their great advantages. They couldn't be so easily brainwashed. You couldn't look in the back of the book for the answer. You couldn't ask the teacher, would that question come up on the examination? and study accordingly, or not study.

These children had the great advantage, in the process of having to copy manuscripts to get their own texts, of being required to understand what they were copying. And by having them copy from manuscripts those things which represented the greatest discoveries, original discoveries in geometry and other subjects, by known people, the child was induced, at the secondary level, to re-experience the act of an original, fundamental discovery in science or art, or statecraft, or theology.

Now, when that happens, you are actually touching, in the child, in education, that which sets man apart from and above the animals: the power of creativity which makes the individual, that potential, in the image of God. Only in the tradition of Moses, and especially of Christianity, does this notion of God exist, and does this notion of man exist. In no other form of society—even though there are inklings of it in ancient Confucian teachings, and even though the idea, from a philosophical standpoint, is developed by Plato—nowhere except in modern European civilization, did there emerge the practical application of the concept of God as a personality of creative intellect, and *man made in the image of God* by this quality, and nowhere else was this ever applied to define the natural rights of mankind, or to apply this as the *governing constitutional principle of society*. And this occurred, essentially, during the fifteenth century.

This idea of the modern nation-state was developed by *geniuses,* geniuses who were produced by this kind of secondary educational method.

In France, of course, the most important feature of Louis XI's reign, apart from the fact that he doubled the per-capita income of France during his reign, was the emphasis upon the assistance of the Brotherhood of the Common Life in creating teaching institutions which reached out to young, pre-adolescent orphans and boys of poor families, to turn these children into, in many cases, geniuses.

As a result of this educational process, there emerged, within the bowels of what had been feudal society, a growing number of persons capable of generating and assimilating and using new ideas, what we call generically scientific and technological progress, but not limited to that. As a result of that, the per-capita income of Europe, and then, through evangelization, of the other parts of the world, began to increase.

No limits to growth

By the time of the fifteenth-century Renaissance, the human population had risen from a potential which is about that of the baboon (several million individuals globally), to about 300 million people, something like that, plus or minus. Today, we have *five and a half billion people,* approximately, on this planet. With the existing technology, fully used, we could easily sustain 25 billion, approximately. We have not yet reached the limit, by any means, of scientific progress. I predict that we can, within 100 years, increase the energy-density beyond that of fusion energy by three orders of magnitude, with matter/anti-matter-related types of controlled reactions. We could do that, if we're determined to do it.

So, there's no limit to man's improvement and growth. What is as significant as the increase in the *number* of people, is the increase in the demographic characteristics of populations.

For example, if we wish to fully educate a young person to the potentials of modern scientific and related knowledge, we have to send them to school for the first years of their life, up to the age of 22 to 25, some more, in certain professions. Maybe we could make it more efficient by a more Classical humanist approach; but nonetheless, this proposition, that we have to keep people in school until they reach the age of 22 or 25, or whatever, is a characteristic of modern technology.

Now, could you sustain a population of students in school to the age of 22 to 25, if the average life-expectancy of the human species were 40 years of age? It would be economically unfeasible. You would have children coming out of school

A Schiller Institute geometry workshop in Boston, June 1995. In the Classical humanist mode of education, students reproduce the greatest discoveries of geometry and other subjects; re-experiencing those creative discoveries fosters in the child that creative power which sets man apart from and above the beasts.

orphans, society unable to pay for it. So therefore, the increase in longevity, the increase in the conditions of health of the population, conquering and eliminating disease through sanitation as well as medical science, are an *essential part* of a decent life, as we understand a decent life for people today.

That did not exist, prior to the Renaissance. So society increased not only in numbers of people, but also in the quality of life for people, in their cognitive qualities, their development as human beings; not only to educate them as human beings, to cultivate the quality of creativity, but, that when they go through school into society, they find professions and employment in a mode which is suitable to an individual who has been developed as one in the image of the living God. That's a new idea.

The development of political economy

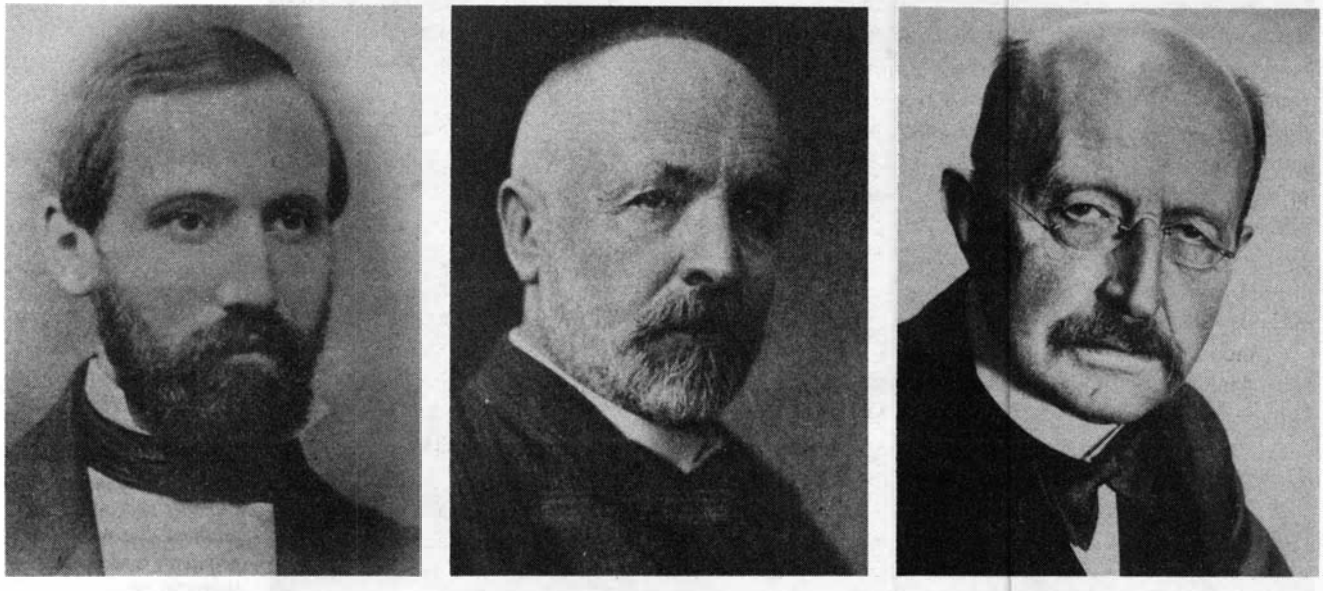
The characteristic feature of this, from an economic standpoint, is the rise of political economy. These changes were brought about not by private enterprise; they were brought about by the state. And the change came largely in the changing of the character of monarchy to a true constitutional monarchy, which the British monarchy, for example, to this day, is not. The British constitution is the power of the royal families and all the laws pertaining to the powers of the royal family. That's the constitution.

But Louis XI had a different conception. He had the conception of a state which would foster education, a state which would build roads, a state which would build canals, a state which would foster trade, a state which would foster improvement in agriculture, a state that would foster science and technology, a state that would foster investment in key industries, a state that would mobilize credit to build infrastructure, to provide credit to new industries.

This was the characteristic of the evolution of society, in its good part, European society, until modern times, until about 1963. They killed Kennedy, they got Adenauer out of power, and they eventually got rid of de Gaulle when he was President. We've been going downhill ever since, with the counterculture.

This produced the higher productivity, the improved standard of living, the fostering of the means of infrastructure to make this possible. It produced a quality which is called *profit,* or if you use the term "macroeconomics," *macroeconomic profit.* This means, essentially, that to maintain this population at that level of existence, to maintain the infrastructure, to maintain the education, to maintain the science, to maintain the sanitation, to maintain that level of technology, requires a certain consumption not only by the population, by the households; a certain consumption by infrastructure; a certain consumption by various kinds of production; consumption by various forms of institutions of physical distribution of goods; and we even have to allow a certain amount to keep the state going, and some services which are quite marginal, which is the equivalent of an overhead expense. It's not productive, but you have to have it.

But this is the input. As we would say in simple, crude



Leaders of the anti-Aristotelian faction in German mathematics and physics, left to right: Bernhard Riemann (1826-66), Georg Cantor (1845-1918), and Max Planck (1858-1947). LaRouche's work on Riemann and Cantor contributed to his breakthrough in economic science; as for Planck, he is probably "the last great scientist who can be said to belong to the tradition of Leonardo."

thermodynamics, this is the energy of the system at that moment, the energy of the system required to keep society from going backwards, from devolving.

Now, what we find is that, over and above the input required to maintain a modern nation-state economy, in healthy conditions of the state, there is a very significant profit. The nation produces more than it consumes of the things which it needs to consume. This is a profit.

Out of this emergence of profit and the emergence of the nation-state, came what was called *political economy*.

We know of four basic kinds of political economy which have appeared in the past 550 years, and we know of one new one which has been invented, which will not last very long, which I'll just identify.

Leibniz and the 'cameralist' tradition

The first notion of economy is that which is typified by Louis XI, by the amanuensis, in a sense, of Louis XI, Jean Bodin, with his *Six Books of the Commonwealth*. Contributions were made in England by certain people. . . .

But then, in the course of the sixteenth century, there emerged an expression of this new idea of statecraft, called cameralism. From about 1671 until the time of his death, Leibniz made a revolution in cameralism, that is, introduced new qualities which had never before existed, going beyond simple cameralism, which is how to increase the profit of society, how to raise the standard of living, these arts and how you look at these, how you measure them.

Leibniz introduced the idea of *power*, that there's a question of *power* in economic progress. There are two kinds of *power* which he dealt with. One was obviously power in the sense of *energy;* and Leibniz was the first to recognize, actually to define, what became known as the Industrial Revolution: that if you burn or use some other source of heat to generate power, you can increase the productive powers of labor in any form of production, by, implicitly, a hundredfold, simply by the application of sufficient power, per capita, to make this possible.

But also, this is a case in which the power is not necessarily increasing, but in which a new technology added—as to make a knife sharper, for example—a new technology added to the structure of production, or something related to that, also increases the productive power of labor, always meaning, that from the standpoint of the society as a whole, the energy of the system exists, but, relative to that energy of the system, *the rate of free energy to energy of the system increases*.

That is, the energy of the system increases, according to Leibniz, as the society develops, as it achieves higher productivity. But if it is done properly, the ratio of the free energy to the energy of the system, also increases.

It was Leibniz's theory of economy which did the most to shape the policies of a new nation in the latter part of the eighteenth century: the United States as a federal republic. What became known as the American System of political economy, as in all parts of the world it was known, as with Friedrich List in Germany, was a product of the influence of Leibniz on the thinking of the Americans who made the federal Constitution. The success of the United States was always based on this principle.

The principles are: development of education, state fostering of infrastructure, the state's creation of money and control and protection of the money, the state's function to protect the farms and industries of its people, to make sure they're able to operate at a profit, not forced to dump their goods on the world market at a loss, and thus to develop the productive powers of labor, in order to create the means wherewith to attack the problems of society. And, despite every up and down, that was the policy of the United States from the time it was adopted in 1789-90, until 1963, when Kennedy was assassinated.

Since that time, the world has introduced the idea of *post*-industrial society, or information theory, and it's gone down. But we'll come to that in a moment.

Oligarchical economics: the Physiocrats

The second model was introduced by an enemy of Colbert. Colbert was a cameralist, and, for a period of time, he was a co-sponsor of the career of Leibniz. In France, there was a group of people who didn't like modern society, who hated Louis XI. These were the people who killed Henri IV, the king of France, and thus made possible the Thirty Years' War in central Europe. They were called in France the *Fronde;* I call them the lunatic *Fronde*.

They were always treasonous to France, they were always Anglophiles, from the seventeenth century on. In the early eighteenth century, they developed a theory called the Physiocratic theory, from which the kind of economic policy you know in governments today, including the communist governments, is generally derived. That is, communism and capitalism have the same mother, exactly the same mother, and she walks the streets at night to support the same family.

The Physiocrats argued that all of these theories deal with the so-called theory of profit: Where does the profit come from which we have in modern society, as it did not exist prior to modern society?

According to the Physiocrats, especially Quesnay, who was a Venetian . . . it is not the farmers, or the miners, who produce the bounty of nature. They are no more than cattle. They are like animals, like cows, from which you extract milk; sheep, from which you extract meat and wool. You are obliged to feed them, as many as you need; you kill the others. You are obliged to allow them housing, so they don't freeze to death, *if you need them*. A page from Gogol's *Dead Souls,* in short.

But they didn't produce the wealth. They are only human cattle. This is what the "neo-conservative movement" says today: "They're only cattle."

Well, to whom does the bounty of nature then belong? "Nature created it," and since France was nominally a Catholic country at that time, they would say, "God." But who gets the bounty? Who has a right to the bounty? Oh, the feudal landlord. Why? Because the feudal landlord has his estate as a gift from God. God has chosen him to become the feudal landlord, chosen his family. Therefore, the bounty of nature belongs to the feudal landlord. So in anticipating Marx, you could say, "The Physiocrats believed in a 'dictatorship of the feudal class,' and attributed all profit to the beneficence of the feudal class, in allowing peasants to work."

Isn't that nice, to become a serf or a slave, get permission from this kind gentleman to work on that estate?

In this connection, politically, [Quesnay] argued (and this was copied directly by Adam Smith), *laissez-faire*. *Laissez-faire* means that the state and urban society must not interfere with the pleasure of the rural landlord, must not regulate rural relationships, must not tax the rural landlord, etc. The king may come and beg for support from the feudal landlord as a gift, but the king may *not* impose, as a king, a tax on feudal wealth.

The Venetian financier nobility

So, along came the eighteenth century, and there came along in Britain a group of people who were not feudal landlords; they were Venetian financier nobility. They had moved up, like body snatchers. They're like people from outer space who come and suck the blood out of the people and take over the people, in the British Isles and the Netherlands. And they believed in financial power; the big families of Venice were known as the financier nobility, not a feudal aristocracy. In England, they assimilated the feudal aristocracy into the ranks of the financial nobility.

One of the most evil leaders of this, was a fellow called the Second Earl of Shelburne, William Fitzmaurice Petty, whose grandfather had founded the Bank of England. Petty had an agent, a lackey, by the name of Adam Smith. Adam Smith was noted for his immorality. And I'll just pause on this to make the point, because it's relevant to the question we're dealing with, with Cantor and Riemann.

In the early eighteenth century, the theories of Galileo and his student Hobbes, were expressed by a fellow, Bernard Mandeville, who wrote a book called *The Fable of the Bees*. The doctrine of this book is that you must not impose morality on people; and the argument he made was that people are naturally immoral, and therefore you cannot make them moral. You must accept and legalize, in effect, their immoralities, because, in the manner that Hobbes describes in his *Leviathan* and so forth, the interaction of conflicting immoralities becomes the Good. This is John Locke's notion of the Good, the "social contract" idea. And [Mandeville's] slogan was, "Private vices, public virtues": Out of the practice of vice among the people, interacting with each other, the conflict produces, asymptotically, the public good.

Now, Adam Smith is famous for this. In his 1759 *Theory of the Moral Sentiments,* he states explicitly: People must act according to their instincts, their sense of pleasure and pain, and must not inquire whether or not these actions they take, under the influence of instinct, pleasure, and pain, work for the good, or for the bad. God is responsible for the outcome

Agents of British intelligence, left to right: Giuseppe Mazzini (1805-72), Friedrich Engels (1820-95), and Karl Marx (1818-83). Marxian economics is one variant of oligarchical ideology, according to which the proletariat secretes profit as an epiphenomenon. Engels was even more absurd, insisting that scientific and technological progress come from "the horny thumb of labor," the opposable thumb.

of evil. That's their argument.

That's also the argument implicit in *laissez-faire,* and in free trade.

Smith went to France, under assignment from his masters in London, France, and the parts of Switzerland where French-speaking evil dwells; from there he developed a parody, in the sense of plagiarism, of the work of, especially, Quesnay, the author of the Physiocratic doctrine. What he did, was to change one axiom in the Physiocratic dogmas. Instead of the "bounty of nature," he introduced the notion of the "bounty of trade"; and thus he created the notion of the dictatorship of the London financier nobility and its merchant class. The nobility are like the queen ants, who send the other ants out to milk the milk-cows and gather the grain. They're called merchants.

The Marxian variant of British economics

The next step in this, also with a very slight change, was Karl Marx. Now Karl Marx didn't know it, but he was a British agent. He didn't wish to know it. He was recruited, initially in Bonn and then in Berlin, to the British intelligence organization created by Lord Palmerston, under the leadership of Giuseppe Mazzini. This was called "Young Germany." And when Marx got into trouble, he went to London, where he was under the patronage of . . . [Mazzini], who was there most of the time he was there, who directly created the so-called First International, and put Marx in charge of it. And Marx remained a protégé of the British intelligence service until 1868, when they decided they'd had enough of him, and they began to dump him, in favor of Bakunin. They finally got Bakunin to eliminate Marx's influence at that point. He continued to write, but he did not have any influence in the world, until he was revived, again by the British intelligence service, through a British agent by the name of Friedrich Engels, in the 1890s, when they decided to unleash Marxism on the world, for geopolitical reasons.

Now Marx, in London, under the direction of David Urquhart—the British Foreign Ministry intelligence official who controlled Marx in London for Lord Palmerston—wrote and developed, by various stages of approximation, a so-called theory of political economy. Marx made two changes in Smith's and Ricardo's theory; otherwise it's the same.

The first change, which is a good one, is that he introduces, in place of individual interaction, social reproduction. In that sense, among those in this series, Marx is the only one among the Physiocrats, among the British economists generally, who actually accepts the idea of macroeconomics. . . . There's a principle of *social reproduction of the society as a whole* which is involved as the determining factor in society. That's the only good part about Marx.

What he did otherwise, was to change, from the bounty of nature and the bounty of trade, to something else. Marx says that all wealth and profit come from the labor of the proletariat. Engels is most explicit on this, later on, as well. Engels insists that scientific and technological progress come from the "horny thumb of labor," the opposable thumb. Engels was not a very good scientist, he didn't know about apes, he only knew about girls, whom he chased a lot. But nonetheless, he believed that man had a uniquely opposable thumb, and he believed that the qualities of the hand, with its oppos-

able thumb, made man capable of using tools, and that man, by random innovation, using the opposable thumb (not for hitchhiking, but for making tools), actually created technology. And you will find that Marxist theory generally, especially among the radical Marxists, especially the anarcho-syndicalists, to this day, will insist upon that form: It is labor, organically, at the point of production, that produces.

What is profit?

So thus, in none of these three types of oligarchical theory—whether the Physiocrats, the landed aristocrats; or the Venetian nobility types, the financier class; or in the case of Marx—is there a *rational,* intelligible explanation of the source of profit. In each case, they resort to a metaphysical argument. They say, in the case of the Physiocrats, that it is the land title given to the nobility by God, which secretes the bounty of nature as an epiphenomenon. They insist, in the case of Smith, that it is free trade, which is nothing but his version of *laissez-faire,* of the production of good from evil; business must be evil, and from that comes good, which is free trade; that's their argument. And from this, is secreted as an epiphenomenon, profit. Marx says no; even though the workers are ignorant, *they* secrete profit and technology as an epiphenomenon. So nowhere is there a rational explanation of profit.

Now you have another variety, which is called "information theory." According to the cybernetic information theory version, which is called the "Third Wave" in the United States, a number of people sitting around, manipulating information, bits of information, like particles in a mechanistic gas theory, somehow "secrete" profitability for the future, as an epiphenomenon.

None of them has any explanation for profit. This is quite similar to exactly the problem that was addressed by Riemann, and, by my stating what I say so far, you begin to see how I came upon this, and [why I have] here expressed, today, my great debt to Georg Cantor.

Economics is key to 'subjective science'

The progress of man, from the most primitive conditions . . . involves a series of discoveries, some of which we know. Language itself is a discovery. The development of language is a discovery. The development of principles in art is a discovery. The development of mathematics is a discovery. Now, in each of these cases, if we know the history, we will attribute each discovery to the name of a person, or a group of persons, because it involves an act of creativity which occurs only within the mind of the individual. Other people can have the same idea, but they have to *re-experience* the idea, the discovery of the idea.

What we call "culture" and "education," particularly when you look at education from the standpoint of the Humboldt educational principles for secondary education—the proper education is to re-experience, in the mind of the child, in succession, the most important original discoveries in art and science of all previous history. Thus children do two things: They acquire knowledge, not just as textbook formulas that they've memorized, but they have *re-experienced the discovery* of the knowledge; and thus, this knowledge is their own. But at the same time, by re-living in the same way an infant relives the discoveries of many generations before . . . the child is also experiencing the *power* of that creative potential which distinguishes man from the animals, that aspect of man which is distinctly in the likeness of God, made in the image of God.

Thus you have an individual who comes out of this, not merely with information, but with *knowledge,* knowledge being not only being able to regurgitate descriptions of what people did in the past, not mere contemplations, but knowledge as a development of the *power* to stand, so to speak, on the shoulders of one's predecessors, and make a new step forward.

It is also the *power* to be able to understand and to utilize what some discoverer gives to us, so we can use it. You cannot take a bunch of people who were aborigines in training, and give them modern machine tools and modern technology. It won't work. You must develop their children. You must give their children the access to all of the best knowledge we have of our culture, and theirs before us. Then they have the power, both to *assimilate* knowledge, and to *generate* it.

Thus, if we look at this matter that Riemann attacked in geometry from that standpoint, we shift our attention away from what is generally taught today: Instead of talking about "objective science," we talk about *subjective science;* and economics is the key to subjective science.

The question is: People can imagine all kinds of things which are contrary to what is generally accepted now. Well, some of these are wrong, and some of these are right. The ability to tell the difference, we call *science.* Any original change in the behavior of the individual in society, which is made on the basis of the imagination; that's your set. Now within that, some of the changes are bad, some are good. What's the difference? The difference is called "science."

The question is whether this discovery of principle can be shown to increase the power and inclination of mankind to improve the condition of mankind, to improve mankind by the standard that man is made in the image of God, and to provide greater power to enable the society to receive each new individual in a way which is consistent with a creature made in the image of God, a society which is not based on hate, but a society which is based on the kind of love which is expressed by seeing behind the eyes of another person, an individual made in the image of God.

Thus we find that certain principles of discovery lead to good results, and certain principles of discovery lead to bad results. Thus we find that geometry is somehow *bounded,* not by imaginary fences in outer space, or strange "warps"

in the space-time manifold; but space is bounded in a different way.

The curvature of space-time

Go back and look briefly at what Riemann was attacking, and what Riemann means by the curvature of physical space-time. He doesn't mean "warp space." "Warp space" is an idea that belongs to warped scientists, it does not belong to healthy ones.

What is the space of the imagination? First of all, the space of the imagination is not true. The universe is not simply extended in a continuous manner, as various people, including Kepler and, later, Max Planck demonstrated. Space is quantified, it's a quantum field. . . . It is not simply extended, nor is it extended with perfect continuity in the very large and in the very small; as Riemann argued, it is no longer necessarily continuous, it is interrupted by discontinuities and singularities.

What do we mean by "singularity"? Well, it's a true singularity in knowledge. We'll get the matter *subjectively,* rather than just objectively. A singularity in knowledge comes how, in respect to geometry? Talk about geometries which are based on different hypotheses, all understood from the standpoint of a *higher hypothesis,* a notion of geometrizing. In a formal system, if you change one of the axioms, you have a new system which cannot be reconciled or derived from the old system. Even though the theorems may pertain to the same subject matter, the two are not consistent.

The difference between the two systems, as in the case of an Aleph series of Cantor's, is an *absolute discontinuity.* There is no way, by chaos or any other way, or various kinds of these figures, of getting across that gap, no matter how small it is. It's not the size of the gap that's important; it's the existence of that gap that's important, that you could never achieve perfect continuity, which is what Leibniz argues in his *Monadology,* which Euler argues against. Euler is absurd; Riemann settles the question.

Therefore, what we're looking at in physics, is our ability to master the physical universe, in terms of a succession of axiomatic changes in the theoretical way we look at the universe and govern our practice. What we're looking for, is a principle which we can use to guide us in making judgments about new products of the creative imagination.

How are we going to know whether a line of work, a line of investigation, is going to lead to a bad result or a good result? We don't know what the good result will be, necessarily, and what the bad result; but we know it will be good or bad. Dino de Paoli has already touched upon this question of *power.*

What methods of higher hypothesis increase the power of mankind as a whole, per capita, over the universe? A measurement of man's ability to survive and improve the condition of humanity, both moral condition, imminently, and the physical condition and demographic condition?

Now, look at the absurdity of simple space-time from this vantage-point, as Riemann would. In simple space-time, you have extension, up, down, backward, forward, sideways, and then, in time, backwards and forwards. But what do you do? It's empty. How can you construct a theory of empty space? Well, you have to put something in it. So you have a sense-perception. You take the sense-perception and you say, "Well, let's put the sense-perception in a point in space and time, where I think this thing occurred. Now, it's not infinitesimal, so it has some sides. Now I have to deal with the *displacement* of space-time by the object. Now I can make simple linear measurements among these objects in motion, and I can make calculations"—all absurd. All absurd. Because space-time is not organized that way.

So what Riemann says, is when we're looking at the relations in physical space-time, we find that we've gone through a succession of discoveries, which leads to the equivalent of a different *curvature* of space-time. It's not that we *see* a curvature of space-time.

An example: the curvature of the Earth

Go back in the history of mathematics, to one case that I often use pedagogically, the case of Eratosthenes, who was very important in terms of some of the work of Cantor, with his sieve construction. But it is also significant for a much simpler discovery, which is estimating the size of the meridian of the Earth [see **Figure 1**]. This is well-known in most decently ordered classrooms; but the significance of this is often overlooked, especially the significance of what I'm discussing here today.

How do you do this? We're doing it again, we're going to have it for children, because I've insisted that these kinds of things from the past, must be organized in a way that children can use them, at the earliest possible [age], as soon as the child is capable of understanding something, to have something ready for them, which is a channel for the development of their mental powers.

Construct, as we are doing. Take a hemispherical shell. Put, along the diameter of the hemisphere, inside, a stick, or a piece of metal. Now, on the other side of it, you can hang a plumb-bob, or some other device, for situating it so that it is aiming at what you think is the center of the Earth: up, down.

Now, take this at two points, A and B, along the meridian. one directly north, or more or less directly north of the other. Now, wait until noontime, which you define by the time the Sun gets into its relatively highest position. The shadow will be cast accordingly. Now, measure the angle which the shadow of the Sun projected by the stick casts upon the interior surface of that hemisphere. A comparison of the two angles, at Point A and Point B, by similar angles, will define the angle of the arc of a circle.

Now if you measure the actual distance from Point A to Point B, you have essentially approximated the measurement

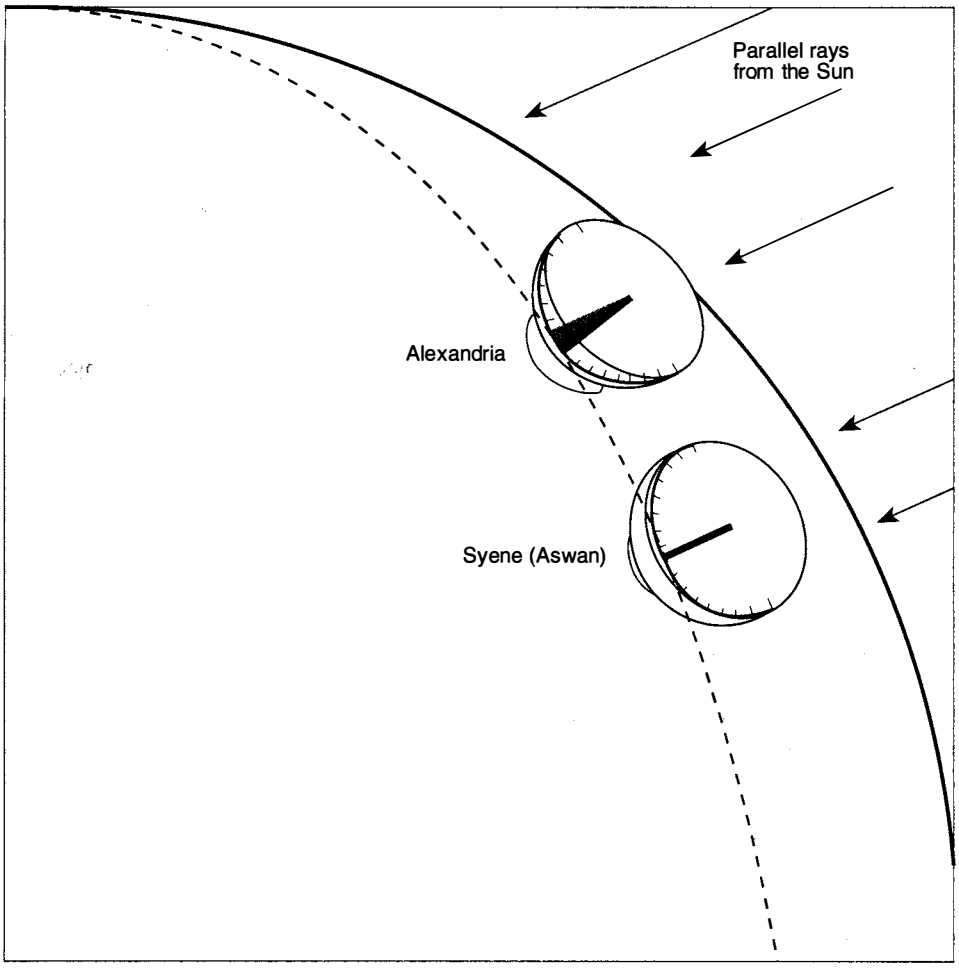


FIGURE 1
Eratosthenes' method for measuring the size of the Earth

In the Third Century B.C., Eratosthenes measured the circumference of the Earth with an accuracy of about 50 miles, even though nobody would see the curvature of the Earth until 2,200 years later. In this illustration of his method, two hemispherical sundials are placed on approximately a meridian circle, in Alexandria and Syene (Aswan), at noon on the day of the summer solstice. The gnomon in the center of each sundial is constructed to be the same length as the radius of the hemisphere; it points straight to the center of the Earth. Eratosthenes found that the gnomon made no shadow in Syene, but a shadow of 7.2° in Alexandria. He also knew the distance between the two cities to be about 490 miles. This allowed him to calculate that the Earth's circumference is about 24,500 miles.

of the perimeter of the circle along that arc; and therefore, by simple construction, ancient people, using the method of Eudoxus, which was used by Eratosthenes, can approximate the size of the Earth.

Eratosthenes was off by 50 miles, in estimating the diameter of the Earth from pole to pole, which, considering the crudity of the methods available to him, *is not bad.*

Now, suppose someone says, "Okay, that's empirical, objective science." No, it is *not* "objective science." It is *subjective* science.

First, ask a question. Okay, the child is being asked to measure the size of the Earth. Has that child ever seen the curvature of the Earth? Did Eratosthenes or anybody who lived before him or in his time see the curvature of the Earth? Of course not. They couldn't.

So therefore, the idea of the curvature of the Earth did not exist as a sensory sense-perception. Therefore, it existed only as an *idea!* And from this idea, a notion occurred.

The same thing in Aristarchus, who said that the Earth orbited the Sun. All the other important discoveries which were made in ancient society, had that character. They involved *ideas;* they involved the use of ideas to influence the development of new ideas.

What we wish to do, is to educate children to understand these *ideas,* in the sense that Plato defines ideas, and defines the relationship between *species,* of types of actions, and ideas as such, Platonic ideas. And that is exactly what Riemann is doing. He is dealing with Platonic ideas.

This was not new to him at that point. Years earlier, before [Johann Friedrich] Herbart died, Riemann, before going to Berlin and then coming back to Göttingen, had attended some lectures at Göttingen which were given by Herbart, who was very much influenced, actually, by [Friedrich] Schiller, when he was studying at Jena earlier, and then had gone on to the No Man's Land where Kant had been to teach, and then was brought back by Wilhelm von Humboldt, for these lectures he gave. . . .

But in the posthumously published works of Riemann, these notes that he made on ideas, metaphysics, and so forth, are included. Riemann refers to these Platonic ideas as *Geistesmassen,* objects which exist only in the mind as ideas, which do not exist as sense-perceptions.

In the latter three types or four types of political economy I discussed, you would have the case of the Physiocrats, the

case of Adam Smith, and then the rest of the British School, or the British School of Karl Marx, or the modern information theory; they allow only the existence of sense-perceptions, and anything else is defined as "attributed epiphenomenon," or attribute, epiphenomenal attribute, of a sense-perception. . . .

Only in this view of science [of Riemann, et al.], are ideas treated as ideas. For the formalist, ideas do not exist, only metaphysical attributions based on sense-perceptions. Every fundamental scientific discovery creates a singularity, and creates it *outside* the domain of sense-perceptions, so that man now has proven knowledge which enables him to *increase* his power over the world of sense-perception, as measured in terms of the development of the number and quality of existence and productivity, of individuals.

How do you measure progress?

The question, then, is: How do you measure these things?

Scientific discoveries and related discoveries in art all have the form of metaphor; the generic form is metaphor. Every important singularity in the theory of knowledge, whether in physical science, in mathematics, or in Classical art, occurs as *metaphor*. The fallacy of information theory, is that you could never put an idea in information theory. Impossible! Because all ideas are metaphors. How could you measure the *power* of an idea according to its statistical characteristics, in terms of an inversion of Boltzmann's H-theorem? You can't do it, it's irrelevant. It has nothing to do with it.

It is not the number of bits of information that counts. Bits of information pertain to communications networks of inanimate objects, of non-living objects, and do not refer to living behavior of *creating ideas,* or communicating ideas. How do we communicate ideas? With metaphor.

What you do, when you communicate with someone, is to demonstrate the existence of a *paradox,* especially an ontological paradox. You identify the paradox by metaphor, as I've used the Goethe "Mailied" as an early form, which, despite the simplicity of the form and its almost trivial content, exemplifies this. It contains the essential thing, which is a metaphor.

Therefore, by communicating *metaphorically,* using the subjunctive and so forth with a language, we can precisely define a singularity, an irony, a metaphor; and, by communicating a metaphor, whether in the science or mathematics classroom, or in the question of tragedy or Classical poetry or music, you thus prompt the mind of the hearer to go through the process of testing the generation of metaphor.

So, how do you measure progress? How do you measure what we must measure, in an economy? What we must measure, is *progress*. You cannot measure progress in terms of some simple Aristotelian-deductive mathematics; how do you measure it? You can't measure it in deductive terms simply, because every time you have scientific progress, you introduce a *discontinuity* into your theory. So therefore, the subject of economic science is not linear algebra, applications of statistics. The subject of economic science, is the *ordering of discontinuities*.

What Cantor indicates as *power,* is exactly appropriate as the word to use, in respect to economic power. The progress of mankind, is what? The progress of mankind is the accumulation of original, fundamental discoveries in what we call science, in what we call Classical art. This is the heritage which each generation passes to the next.

What is this heritage? This heritage is a mass of discontinuities, a mass of discoveries, original, axiomatic quality of discoveries, whether in language, the use of language, or in anything else. What we pass on, is not measurable knowledge, in the ordinary sense; we pass on *discontinuities*. We call this an education, putting a child through original discoveries, re-experiencing them.

So therefore, in economics, the same thing. What defines economic progress? What defines scientific progress? The development of discontinuities. How is this measured? As the *increase in the density of discontinuities in terms of any measurable unit of action in the economy as a whole*.

So therefore, what is the significance of the Aleph series in Cantor? Its application is as an approach to the understanding of the measurement of discontinuities. Now, those who understand Cantor somewhat better, even if imperfectly, will always emphasize that, the fact that there's the implicit denumerability of a finite number of discontinuities within any arbitrarily chosen interval of action. And that's what we're doing.

Now, how do you run society on that basis? Mathematically? Not exactly. What you do, is what I've learned to do. You say, what is it that we commonly believe today, say in physical science, which is wrong? How do we define that? Let's take astronomy. You go talk to the astronomers, and find out what stellar objects or events in the astronomical domain, are anomalous, things which defy interpretation according to existing generally accepted notions of astronomy. And therefore, you say, "Well, it is important that the state consider how we're going to bring about a study of these anomalies, because we know, that by mastering these anomalies, we will correct our existing knowledge, and therefore we will increase the power of productivity of man over nature."

Take it in microphysics. It's simple, you just keep going down the scale, smaller and smaller. Every time you find an anomaly, *that's* what you must concentrate on. How do you know what the benefit is going to be? You don't have to know. Because the question of *policy,* the fundamental question of science, is not whether you can measure something in advance. Some things you can, some things you can't. The important thing to know is: What is the next step you must take? It's like walking through a swamp full of quicksand. You don't have to know how much quicksand there is, how big the stones are, how many stones there are; all you have

to know is: Can you find a stone on which to walk, which will allow you, step by step, to get safely out of that swamp?

And therefore, what we need, is an understanding of this subjective aspect of science, in terms of the notion of discontinuities. A good mathematics is one which is not exaggerated in its importance. Mathematics is an engineering tool which must be constantly improved; but it's always wrong, because the next anomaly is going to overturn it.

So therefore, the important thing to do is to put mathematics in its place, as a little fellow here, who's carrying the bags for the big fellow, science. And science consists in the principles of discovery, and knowing how to make these discoveries, not in being able to pre-measure. Because you won't know how to measure them, until you make them. Then you will find out how to measure them.

So therefore, the question is, the relative power of mankind achieved by what? By . . . anomalies.

What is an anomaly? If there are hungry people in the world who should not be hungry, that's an anomaly. That may not involve a great, new discovery, but that means the *existing policy of practice of the relevant institutions, is wrong*. If the death rate increases, and we can attribute the death rate to some cause, like cutting of pensions, for example, or cutting the health care provisions under existing law, that's wrong, that's murder. If you cause a change in public policies or insurance policy which you should have known would have increased the sickness rate or the death rate, then you personally are responsible for every person that becomes sick or dies as a result of your innovation, contrary to Adam Smith. Evil produces evil, and evil is accountable for evil; and negligence, in that sense, is evil.

So we have to change the focus to the subject, and the important thing is that, as Riemann says, to make an advance in mathematics, you must step outside of mathematics, into the realm of physics. If you enlarge that, as I do, you will say, "Yes, this is true; but let's go one step further, into the physical economy, the process of reproduction of the society, which must become more suitable to the individual made in the image of God, both in his education, his knowledge, his responsiveness, and his accountability for the results."

Notes

1. Georg Cantor, *Grundlagen einer Allgemeinen Mannigfaltigkeitslehrer;* first English translation by Uwe Parpart, "Foundations of a General Theory of Manifolds, A Mathematical-Philosophical Study in the Theory of the Infinite," *The Campaigner,* January-February 1976.

2. "Mitteilungen zur Lehre vom Transfiniten," in Ernest Zermelo (ed.), *Georg Cantor: Gesammelte Abhandlungen mathematichen und philosophischen Inhalts* (Berlin: Springer-Verlag, 1990). The title can be translated as "Communications on the Theory of the Transfinite"; there is no published English translation.

3. *Beiträge zur Begründung der transfiniten Mengenlehre;* English translation by Philip E.B. Jourdain, *Contribution to the Founding of the Theory of Transfinite Numbers* (New York: Dover Publications, 1955).