On the curvature of physical space-time

In 1696, the mathematician Johann Bernoulli issued a challenge to the scientific world, to solve the following problem: "To determine the curve joining two given points, at different distances from the horizontal and not on the same vertical line, along which a mobile particle acted upon by its own weight and starting its motion from the upper point, descends most rapidly to the lower point." Or, expressed another way: "If the curve is replaced by a thin tube or groove, and a small sphere placed in it and released, then this [sphere] will pass from one point to the other in the shortest time." This curve, he called the **brachistochrone,** from the Greek words for "shortest time" (**Figure 2a**).

The curve in question, Bernoulli discovered, was the cycloid (**Figure 2b**)—a curve which had been investigated earlier by Christiaan Huyghens (1629-1695), and described in his book *The Pendulum Clock.* Huyghens determined that a weight falls along a cycloidal path in the same amount of time, no matter from what point on the cycloid it begins its motion. This curve, he called the **tautochrone,** from the Greek for "same time" (**Figure 2c**).

Bernoulli described his amazement, when he discovered that the two curves were the same: "But you will be petrified with astonishment when I say that precisely this *cycloid,* the *tautochrone of Huyghens,* is our required *brachistochrone.*"

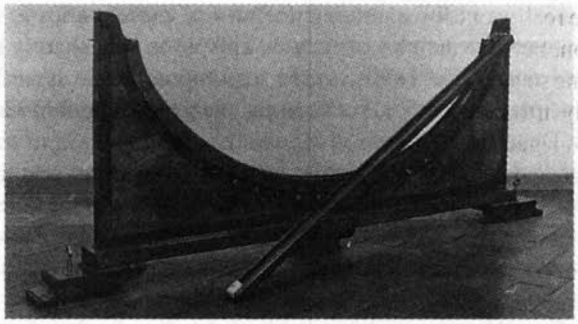
His amazement did not stop there. Bernoulli went on to write that the same property also applied to the refraction of light (**Figure 2d**): "I discovered a wonderful accordance between the curved orbit of a ray of light in a continuously varying medium and our *brachistochrone curve.* . . . The *brachistochrone* is the curve which would be traced by a ray of light in its passage through a medium whose rarity is proportional to the velocity which a heavy particle attains in falling vertically. For whether the increase in the velocity depends on the nature of the medium, more or less resistant, as in the case of the ray of light, or whether one removes the medium, and supposes that the acceleration is produced by means of another agency but according to the same law, as in the case of gravity; since in both cases the curve is in the end supposed to be traversed in the shortest time, what hinders us from substituting the one in place of the other? . . .

"Thus I have with one stroke solved two remarkable problems, one optical and the other mechanical; . . . I have shown that the two problems which are taken from entirely distinct fields of mathematics are nevertheless of the same nature."

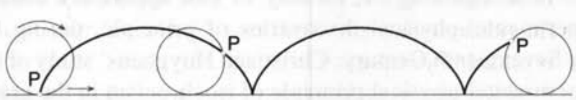
(Quotations are from "Bernoulli on the Brachistochrone Problem," David Eugene Smith, ed., ***A Source Book in Mathematics*** [Mineola, N.Y.: Dover, 1959], pp. 644-655.)

—*Susan Welsh*

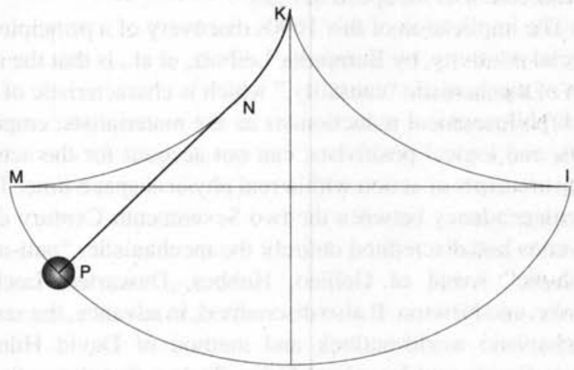
FIGURE 2



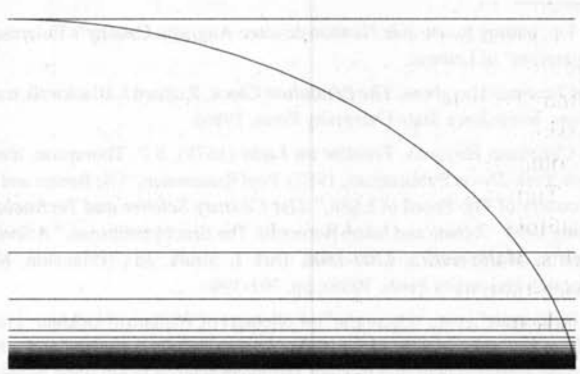
(a) A brachistochrone model built by Francesco Spighi in the 17th Century. A ball that rolls down the cycloidal track reaches the bottom faster than one rolling down the straight track.



(b) The cycloid is the curve traced out by a point on a circle, as the circle rolls along a line.



(c) The tautochrone: Huyghens used the cycloid to make a pendulum clock, because no matter how wide the swing, the time of the swing remains constant.



(d) Bernoulli proved that the cycloid is also the path taken by a ray of light passing through a medium of constantly increasing density.