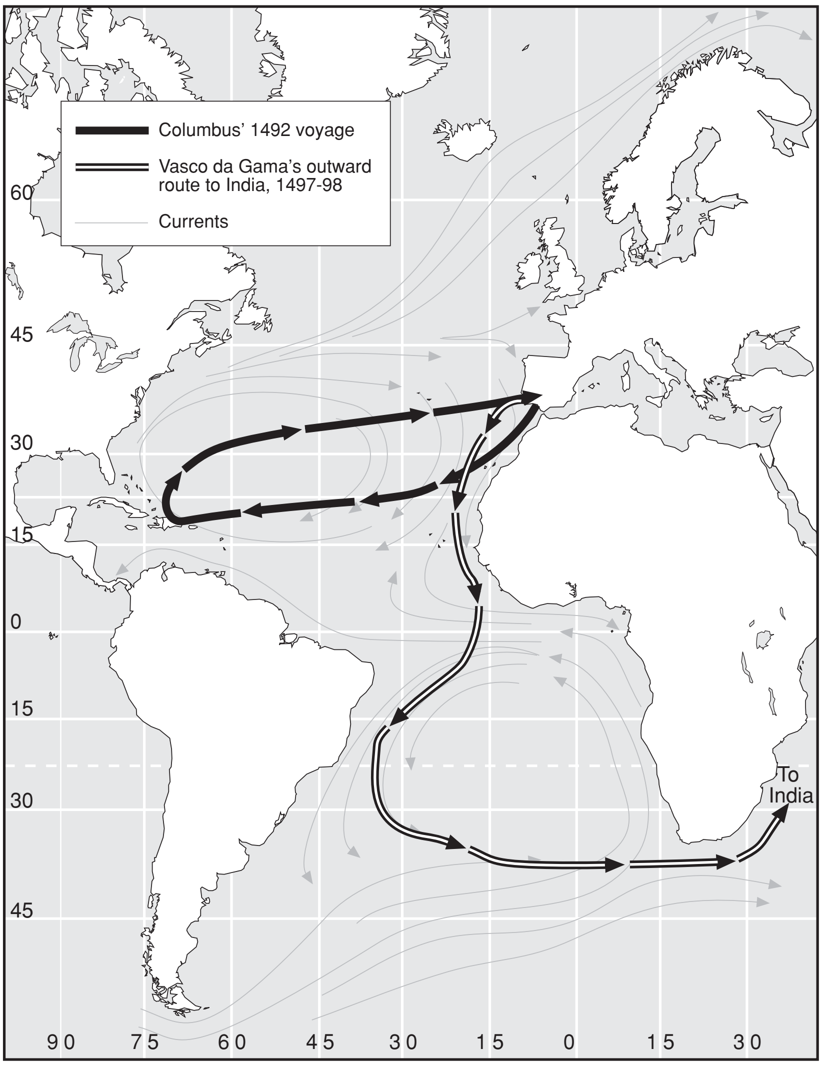
duce. Columbus, who had arrived in Portugal in 1474 and spent 16 years immersed in the most feverish period of Portugal's scientific and nautical breakout, built on the "long-ocean tack" techniques of Henry's captains, to sail "out" to the New World on a southerly route that picked up the westward-flowing tradewinds, and returned on a more northerly route that captured the reverse flow.

Five years later, in 1497, Vasco Da Gama hitchhiked the mirror-image southern-hemisphere circulatory patterns, to turn the Cape of Good Hope and reach India (see **Figure 5**). His "detour" almost to the Brazilian coast, involved being out of sight of land for over three months and 3,800 miles (compared to Columbus's 33 days and 2,000 miles), but it cut the time of the passage in half.

FIGURE 5
The wind and ocean currents used in Columbus's and Da Gama's voyages of discovery



Eratosthenes' Sieve

by Bruce Director

One of Eratosthenes' most important discoveries, was his unique method for finding the prime numbers, now known as the "Sieve of Eratosthenes." Among the whole numbers, there exist unique integers known as prime numbers, which are distinguished by the property that they are indivisible by any other number except themselves and 1. Thus, 2, 3, 5, 7, and 11 are all examples of prime numbers. Numbers such as 8, 9, and 10 can be evenly divided by other integers and are thus called composite.

Eratosthenes' method of finding the primes functions exactly like a sieve, in which the composite numbers fall through the "mesh," and the prime numbers remain. The "mesh" in this case, is the ordering principle by which the composite numbers are generated from the primes. To this day, Eratosthenes' method is essentially the only one for finding the prime numbers. More important, his approach of investigating numbers in characteristic classes, instead of one by one, establishes a crucial method for scientific investigation. This method was later applied in the physical domain by Gottfried Leibniz and Carl Gauss, and laid the basis for Georg Cantor's later development of transfinite numbers.

Greek scientists prior to Eratosthenes had investigated prime numbers, and Euclid (ca. 300 B.C.) recorded that knowledge in the *Elements*. Euclid showed that all numbers are either prime or composite, and that any composite number is divisible by some combination of prime numbers.

You can prove this for yourself, in the following way: Any composite number can, by definition, be divided by some other number, and that other number is either another composite number or a prime number. If it is a prime number, we need go no further. If it is a composite number, then that new composite number can be divided by another number, which is either a prime number or a composite number, and so on. By this method, you will eventually get to a prime number divisor.

For example, 30 is a composite number, and can be divided into 2, a prime number, and 15, a composite number. In turn, 15, can be divided into 3, a prime number, and 5, also a prime number. So, the composite number 30 is made up of, and can be divided by, prime numbers 2, 3, and 5.

Euclid also proved that the number of prime numbers was infinite. Gauss was the first to prove (*Disquisitiones Arithmeticae,* Article 16) that a composite number can be decomposed into only one combination of prime numbers. In the above examples, no combination of prime numbers other than 2×2×3 will equal 12. Likewise for 504, or any other composite number.

This remarkable result, which Gauss says was "tacitly supposed but had never been proved," provokes a fundamental question concerning the nature of the universe. The fact that Gauss was the first to consider this result important enough to prove, is another indication of his genius, and shows him to be a true follower of Eratosthenes.

Who was Eratosthenes?

Eratosthenes (c. 275-194 B.C.), perhaps the greatest scientist of the Hellenistic world, was also one of its most prolific and versatile: His work included investigations in astronomy, geography, geodesy, poetry, music, drama, and philosophy.

Born in Cyrene, he was educated in Alexandria, Egypt, and Athens by followers of Plato. At the age of 40, he became the head of the famous library at Alexandria, where he remained until his death.

In addition to his measurement of the Earth's circumference, Eratosthenes was the first to measure the angle of the Earth's tilt on its axis (the plane of the ecliptic). He also wrote "The Duplication of the Cube," and "On Means," which were treatises investigating the crucial mathematical paradoxes arising from the investigation of dimensionality. His work "Platonicus" deals with the mathematical and musical principles of Plato's philosophy. He published maps and works on geography and chronography.

Eratosthenes was also a poet, dramatist, and philologist, writing several poems and plays, only fragments of which survive, and a book on comedy. Other ancient writers attribute to Eratosthenes books on philosophy and history.—*Bruce Director*

Exemplary of the singular nature of prime numbers, is that there is no regular distribution of them, and no simple, linear formula for finding them. That is, in any given interval of whole numbers, no matter how big or small, the prime numbers could be anywhere.

How the sieve works

The way Eratosthenes' sieve works is this: List the integers from 1 to any other arbitrary integer, A. Now, beginning with 2 (the first prime number after 1), strike from the list all numbers divisible by 2, for they are composite numbers. Do the same with all numbers divisible by 3, the next prime number; then those divisible by 5, etc., until you come to the first prime number whose square is greater than A. (If A=100, then you only need do this procedure with primes less than 11.)

With Gauss's proof, and the preceding discussion, it is shown that prime numbers are those from which all other numbers are composed. The primes are primary. The word the ancient Greeks used for "prime," was the same word they used for "first" or "foremost."

This raises the question: What happens when you try to construct all integers from the primes alone? First, you'd make all the integers composed only of 2, such as 4, 8, 16, ... Then you'd make all the integers composed only of 3, and of combinations of 2 and 3, such as 6, 9, 12, . . . , and so forth; then with 5, etc. As you can see, this process would eventually generate all the integers, but in a nonlinear way.

Compare that process with constructing the integers by addition. Addition generates all the integers sequentially, by adding 1, but does not distinguish between prime numbers and composite numbers.

The unit 1 is indivisible, with respect to addition. With respect to division, the prime numbers are indivisible. Both processes will compose all the integers, but that result coincides only in the infinite. In the finite, they never coincide. The difference is between the mental act of addition, and the mental act of division. Don't try to resolve the matter, by asking if division is superior to addition. Instead, reflect on that which is different between the two processes, the "in-betweenness." It is the relationship between the numbers, which is the object of our thought, not the numbers in themselves.

This anomaly is a reflection of the truth that there exists a higher hypothesis which underlies the foundations of integers—a hypothesis which is undiscoverable if limited to the domain of simple linear addition. By reflecting on this anomaly, we begin, as Socrates says, "to see the nature of number in our minds only" (from Plato's *Republic*). Our minds ascend, as Socrates indicates, to contemplate the nature of true Being.

We ask, "If the domain of primes is that from which the integers are made, what is the nature of the domain from which the primes are made?"