III. Review

BOOK REVIEW

Pierre Fermat: Ending the Tyranny of Descartes and the Cartesians

by Pierre Bonnefoy

The Battle for Light: Fermat vs. Descartes
A Sourcebook on Pierre de Fermat's Principle of Least Time
Jason Ross, editor and translator
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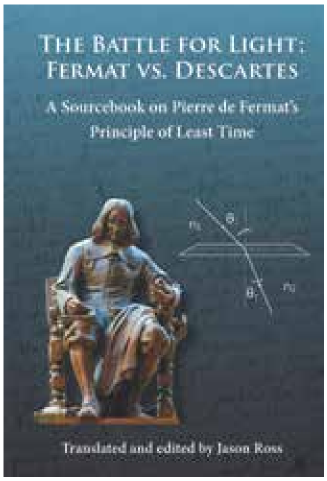
Dec. 12—*We publish here, with permission, the afterword to Jason Ross's* The Battle for Light: Fermat vs. Descartes, A Sourcebook on Pierre de Fermat's Principle of Least Time. *Pierre Bonnefoy's afterword is a masterful summary of the book that goes beyond a mere review, situating Fermat's work amidst the battle within science at the time and laying out how later discoveries by Christiaan Huygens and Gottfried Leibniz built upon Fermat's scientific breakthroughs.*

The Battle for Light *is the first comprehensive English translation of Fermat's complete correspondence on light, featuring his landmark disputes with René Descartes, whose explanation of refraction Fermat found totally unconvincing. It presents Fermat's complete correspondence on light, alongside the* Dioptrics *of René Descartes and the work of Marin Cureau de la Chambre, whose idea of "least distance" catalyzed Fermat's formulation of least time. Ross's translations, presented along with their historical context, offer a fascinating glimpse into the debates that shaped the progress of physical science. Ross's sourcebook also includes Fermat's groundbreaking work on finding minima and maxima—which allowed him to precisely calculate the angles of refraction generated by his principle of least time, and which was a key inspiration to Leibniz's development of the infinitesimal calculus.*



When my friend Jason Ross invited me to write the afterword for his book on a fundamental philosophical and scientific battle that took place in my country, France, nearly four centuries ago, I accepted with great pleasure, because this task allows me to highlight, in passing, that not all French people deserve the bad reputation sometimes attributed to them—that they are all Cartesians. On the contrary, some of my illustrious compatriots have fought against Cartesianism, as we have seen in the preceding pages.

The Historical Context: Fermat vs. Descartes

The quarrel between Pierre Fermat (1601-1665) and René Descartes (1596-1650), which Ross has unveiled in this work by relying on original texts, represents an important historical episode that made 17th-Century France the beacon of science in Europe. This was notably reflected in the establishment of the Académie des Sciences in Paris in 1666 by Jean-Baptiste Colbert (1619-1683). More intelligent than many of today's Western politicians, Colbert appreciated the value of uniting scientists globally, transcending political divides.

However, merely gathering scientists together does not guarantee a harvest of discoveries; there must also exist, in their mutual interactions, an honest search for truth. This requires a certain courage—courage to stand up when one realizes that all one's colleagues are mistaken on an issue. Being honest in such an environment means daring to confront the sacrosanct "scientific consensus."

In reality, this problem arises in all times and places—including our own era. But it is unlikely that

French science could have played its leading role had not a few particularly courageous individuals, among whom Fermat stands as a pioneer, dared to attack the scientific dogmas imposed by Descartes and the Cartesians. Among these individuals, we must especially mention Blaise Pascal (1623-1662), who maintained a friendly and scientific correspondence with Fermat, although they never met; the Dutchman Christiaan Huygens (1629-1695), who directed Colbert's Académie des Sciences; and the German Gottfried Leibniz (1646-1716), who, by building upon the discoveries of his predecessors, achieved a scientific revolution by inventing the differential calculus.

portrait by Robert Lefevre

Pierre Fermat (1601-65)

Cartesian Deduction vs. Creativity

These men's opposition to Descartes was not limited to refuting scientifically false Cartesian theories, but concerned a more profound difference in the approach employed by the researcher to discover the unknown. The fundamental problem of Cartesianism lies not in its specific theories but in its *method* of thinking.

Probably convinced that he had made great scientific discoveries, Descartes wrote his *Discourse on the Method* to show how he had proceeded, doubtless hoping to inspire his disciples to become great discoverers themselves. At the heart of this method is deduction. Descartes asserts that the scientist must first base himself only on truths so evident that they cannot be doubted. Each complex problem must then be decomposed into simpler parts, solved individually through chains of deductions that start from previously admitted or demonstrated truths, to arrive at a solution. Descartes claimed to discover the laws of the universe through reasoning alone, without the need to test theories against experience.

Did he know that his method condemned him to discover only what was already known or assumed to be known? In optics, the mathematical law that describes the refraction of a light ray passing through different media—the sine law of refraction—is still sometimes known as the "Law of Descartes"; yet Ross's book shows us that if Descartes really had discovered this law, we could deduce that he was not a practicing Cartesian himself! Indeed, the "explanations" he provided for the phenomenon are so fanciful that we understand them to have been added afterward to an already established mathematical law: he did not arrive at the correct mathematical formula by deduction from evident truths, as his method prescribes. We must conclude that the sine law of refraction was probably established by Willebrord Snell (1580-1626), whom Descartes had known in Holland; Snell could not claim it for himself, because he died before Descartes published his *Dioptrique* (Dioptrics), the first book to state it.

Fermat, in contrast, did not practice Cartesian deduction but, rather, the experimental method.

This consists of first formulating a general hypothesis and then testing it through a new experiment: If the experiment's results contradict the hypothesis, then the hypothesis must be abandoned and another sought; if the experiment confirms the hypothesis, then it will be retained to develop a new theory. Of course, this new theory will be adopted only as long as it is not in turn invalidated by another experiment. In other words, the experimental method is based on nothing evident in itself—only on the hypothesis imagined by the scientist—but it constitutes a real wager on the future: The scientist hopes that the result of the experiment will conform to what his hypothesis predicted.

The Principle of Least Time

In the case of light, Fermat hypothesized his principle of least time, according to which light minimizes the time taken to travel from one given point to another. Fermat found that this hypothesis was in agreement with the sine law of refraction, already verified experimentally, which gave him significant weight against the Cartesians, who had no credible theory of their own. It is interesting to note that the experiment that definitively validated Fermat's hypothesis was carried out long after the deaths of both Fermat and Descartes. According to Fermat, light travels more easily and faster in less dense media and more slowly in denser media, whereas for Descartes, the opposite was true. Since 17th-Century technology could not compare the speeds of light in different media, Fermat left it to future generations to decide between him and Descartes. In the 19th Century, Foucault's experiment ruled in favor of Fermat: light is indeed faster in air than in water.

However, the Cartesians had imposed such intellectual terror on the scientific community that Fermat's victory in optics was not sufficient to overthrow their hegemony.

portrait by Frans Hals, 1648

René Descartes (1596-1650)

Pascal's Experiments on the Vacuum

Other victories were necessary, notably including that of Pascal concerning his experiments on the vacuum. In this case, as in the previous one, Descartes's deductive method proved incapable of accounting for a phenomenon incompatible with the old theories.

Consider a transparent test tube, at least 760 mm long, completely filled with mercury. It is then inverted so that its opening is immersed in a basin also containing mercury. In this experiment, which foreshadows the invention of the barometer, the mercury is seen to descend in the test tube to form a column that reaches a certain height above the surface of the mercury in the basin. The question that then haunted scientists was: "What is in the tube between the top of the mercury column and the end of the tube?"

A controversy arose through correspondence between Pascal, who had conducted a series of experiments including this one, and a Jesuit father named Étienne Noël, who had been a teacher of Descartes and contested Pascal's interpretation of his observations.

Noël, like Descartes, based his arguments on an apparently incontestable truth: "Nature abhors a vacuum." Therefore, there could be no vacuum in the tube above the mercury, and thus the space must contain a substance of some kind. Now, it was "known" at the time that there existed only four elementary substances: earth, water, air, and fire. Therefore, the substance in the tube could only be some compound of these substances, or one of them. Therefore, this substance must be air. Yet, this air offered no resistance to compression, since by sufficiently tilting the tube, one could see it give way to the mercury until it disappeared, and see it reappear when the tube was set vertical again. Therefore, this air must be different from air we breathe, especially since the air we breathe could not pass through the glass wall of the tube. Therefore, concluded Noël (and Descartes), this air must be sufficiently "subtle" to be able to pass through small pores that must necessarily exist in the glass.

A perfect deduction from false premises! Just like the correspondence of Fermat on optics that Ross presents to us in his book, the correspondence between Pascal and Noël is so comical that it truly deserves to be translated into English.

Why did Noël refuse to accept that the space in the tube was empty? Because, he said, light obviously passes through the glass and through the space from which the mercury departed. Therefore, there could not be nothingness at that place, for nothingness cannot have a physical property like allowing light to pass. Pascal tried in vain to explain that the vacuum and nothingness are not the same thing, even though the vacuum is not a substance and its true nature remained to be discovered, but Noël would not acknowledge that this new entity could not be identified with an existing one.

The great Descartes also failed to impress the young Pascal, who later noted in his *Pensées* (*Thoughts*): "Descartes useless and uncertain."

Contributions of Pascal, Huygens, and Leibniz

At the end of his book, Ross has wisely included texts by Fermat on his method for finding maxima and minima, which foreshadowed the differential calculus that Leibniz would later invent. I would add that Pascal, for his part, facilitated the use of the method of indivisibles, which foreshadows Leibniz's integral calculus (the reciprocal to differential calculus). To promote the indivisibles, Pascal launched a competition in which he challenged the geometers of his time to solve a series of problems concerning the *roulette*, a geometric curve now known as the cycloid. Solving these difficult problems required mastering this kind of calculation. Some scientists produced original solutions, including the young Huygens. Pascal published his own in his *Treatise on the Roulette*.

Fighting Cartesianism was undoubtedly a difficult ordeal for Huygens, for his father had been a personal friend of the famous French philosopher, whom Christiaan had been raised to hold in respect. Nevertheless, he realized fairly early that the various laws that Descartes had stated for the physics of elastic collisions were inconsistent with each other, and almost all were shown to be false.

To refute them, Huygens developed his own principle of relativity. According to this principle, the laws of physics are the same for an observer standing on the bank of a river and for an observer moving at a uniform speed aboard a boat on the river. Starting from this hypothesis and from the only accurate law Descartes put forward (that when two identical balls moving towards each other at the same speed collide, they rebound after the collision in opposite directions at the same speed), Huygens discovered the law of elastic collisions for the general case (in which the two balls are not identical and their speeds are different).

When he later wrote his *Treatise on Light*, Huygens put forward the hypothesis, validated long after his death, that light is a wave. This hypothesis obviously has nothing to do with the balls and rackets conjured up by Descartes to describe light, as we see in this book. But, more importantly, it allows us to derive the sine law of refraction from the hypothesis that light travels faster in air than in water, which supports Fermat's principle of least time.

Leibniz and the Principle of Least Action

When Leibniz arrived in Paris in 1672, it was with the intention of associating with the best scientific minds of the time to learn mathematics from them and thus become a great scholar. The gamble paid off. Three years later, he invented the differential calculus, which revolutionized all of science. At the Académie des Sciences, he received from Huygens the heritage of Fermat and Pascal, which he fruitfully built upon.

In 1684, he published in the *Acta Eruditorum* the founding text of his new calculus: "*Nova methodus pro maximis et minimis*" ("New Method for Maxima and Minima"). To illustrate the power of this invention through a concrete application, he showed that, from the hypothesis that light propagates according to Fermat's principle of least time, one can derive the famous sine law of refraction in just a few lines, in a way that is remarkably similar to Fermat's analytical demonstration.

With this article, Leibniz implicitly took an anti-Cartesian stance. Starting from the publication in 1686 of his article "*Brevis demonstratio erroris memorabilis Cartesii*" ("Brief Demonstration of a Notable Error of Descartes"), he attacked Descartes's physics more openly and adopted Huygens's approach on the question of elastic collisions. He showed that if Descartes's laws of motion were correct, perpetual mechanical motion would be possible—an absurdity that even the Cartesians could not accept.

In the following years, he engaged in a devastating refutation of not only the scientific theories of Descartes but especially the metaphysical system of Descartes upon which the theories were based.

Generalizing the principle of least time that Fermat had hypothesized for his study of light, Leibniz established the universal principle of least action, which he stated, for example, in his *New Essays on Human Understanding*, as follows: "Nature acts by the shortest ways, or at least by the most determined ones." Thus, the combined efforts of Fermat, Pascal, Huygens, and Leibniz liberated scientific thought from the Cartesian deductive prison.

The Mental Dead-End of Induction

However, another mental dead-end arose at the same time: after deduction came induction. This latter method, seemingly diametrically opposed to the former, was developed in England by the empiricists Francis Bacon (1561-1625), John Locke (1632-1704), and, most significantly, Isaac Newton (1642-1727).

Unlike deduction, induction is not based on "evident truths" but on "objective facts." According to this doctrine, scientists must make a very large number of observations of natural phenomena, while forbidding themselves to have preconceived ideas on the subject. Only after gathering an extensive amount of data should the scientist attempt to state general mathematical laws that allow the prediction of observations, of data. Here, seeking the *causes* of phenomena is forbidden; Newton would famously declare, "I frame no hypotheses."

In reality, induction is just as sterile as deduction. Firstly, because there is no such thing as an "objec-

Courtesy of Jason A. Ross

Ross explains how Fermat's 17th-Century discoveries challenge crucial assumptions about natural processes and scientific method that you have been taught. Here, editor and translator Jason A. Ross at the Fermat Museum in Toulouse, France, November 2024.

tive fact," or an observation made independently of the observer's way of thinking. The empiricist who observes thus makes hypotheses, whether he is aware of it or not, and therefore finds himself *a priori* unable to adhere to his own philosophy. But there is a greater issue: True scientific revolutions occur when individuals have the courage to reject commonly accepted self-evident truths, which requires formulating new hypotheses. Empiricism, therefore, is not the experimental method that explicitly employs hypotheses, as we previously described. A physical hypothesis goes beyond "saving appearances," to state the cause of what is observed.

It is by *hypothesis* that the experimental method studies the causes of phenomena, while empiricists, deprived of hypotheses, must content themselves with seeking to predict effects. Yet, although effects are observable, causes generally are not. Therefore, through simple induction, one cannot ascend to these principles, such as that of least time or least action, which have revolutionized science beyond the mere question of the refraction of light.

This issue, which is significant for contemporary science, warrants further argument, but this would go beyond the scope of this afterword. I will therefore conclude with some remarks on the myth of Newton as it relates to the work of Fermat and his successors.

The Myth of Newton

In his *Opticks,* Newton puts forward a corpuscular theory of light, which constitutes a real regression from the works of Huygens, with which he was familiar. Yet Newton's theory dominated the 18th Century. One of its notable tenets is that light moves faster in denser media and slower in less dense media.

Seeking to undermine Leibniz's influence, Newton claimed that Leibniz had stolen the invention of differential calculus from him, and launched a veritable trial against Fermat at the Royal Society in London—of which Newton was the president! Today, historians recognize the dishonesty of this trial, as Newton was both judge and a party to the affair, but they nearly all say that Newton and Leibniz independently invented differential calculus: an absurd scientific consensus that is easy to refute. Newton never produced a differential calculus; he produced the "calculus of fluxions," which no one ever used outside of a few scholars of the Royal Society of his time, because it is in fact impracticable. Even British scholars had to adopt Leibniz's differential calculus.

Conclusion

What relevance does all this have for us today?

Today, many scientists may struggle to differentiate between the mathematical or computational models they use and genuine physical hypotheses, such as the principle of least time. It is certainly legitimate and even necessary for an engineer to use models in his work. But this is a problem in the case of a researcher who explores the unknown. Models have past theories as their foundation and will never yield anything new. The excessive reliance of today's scientists on modeling (in quantum physics, economics, climatology, biology, etc.) shows that induction/deduction and empiricism, despite their intrinsic limits, have not really been eradicated. To solve this problem, we must study the history of science from original texts and through the original controversies, and not only by reading academic commentaries.

We must draw inspiration from scholars who, like Fermat, overturned established knowledge through revolutionary hypotheses.

This is the approach that Ross, through his work, has invited us to take.