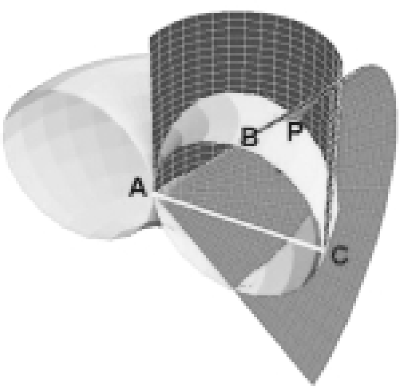
Box 3

The Torus and the Sphere

The sequence that represents the volumes of those cubes that a person can build with unit cubes is 1, 8, 27, 64, etc. In the 4th Century B.C., Plato challenged his collaborators to solve an old problem: Build a cube of volume 2. In other words, construct two cubes, one of which can contain exactly twice the amount of material as the other. This means we must find an intermediate cube, not in the sequence of cubes which are generated by unit cubes.

Hippocrates of Chios had demonstrated that each of the normal cubic numbers in the sequence can be arrived at by a process of geometric growth, in which there are two geometric steps mediating the growth from 1 to the next highest cubic number. For example, doubling produces 1, 2, 4, and 8; and tripling produces 1, 3, 9, and 27. Between each pair of extremes (1 and 8, or 1 and 27) are two geometric means (2 and 4, or 3 and 9, respectively). As a cube doubles from 1 to 8, the edge lengths grow from 1 to 2. But, the two geometric means between 1 and 2 cannot be found on a ruler. In fact, the best one can get by today's calculations, is a close approximation.

FIGURE 1



Computer graphic representation of Archytas' construction.

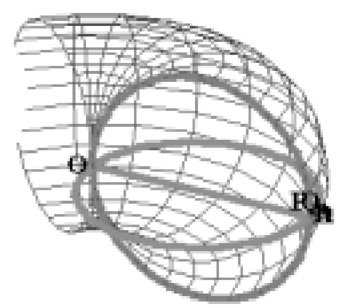
Plato, however, did not ask for a close approximation! Archytas, a close collaborator of Plato, discovered the first exact solution to the problem (**Figure 1**). Archytas *knew* his discovery would produce a doubled cube, because it solved the general problem as posed by Hippocrates. Thankfully, there is a description of Archytas' construction which we can use today to replicate his ancient discovery, by means of the method of *Sphaerics.*

For a youth growing up in the 21st Century, educated inside universities run by Baby Boomers, it's easy for us to believe that Archytas never built his construction. But, this is simply because we've been brainwashed to ignore the process of production, as a human activity. Most members of the LaRouche Youth Movement *have* built contraptions demonstrating different aspects of the actions in Archytas' construction. In the photo, LYM members in Los Angeles use their Archytas model in a classroom/workshop.

To our knowledge, however, nobody has yet actually built a complete model of the torus, the cylinder, and the cone, all intersecting at the cubic point. The difficulty lies not in constructing the cone or the cylinder, but in constructing the torus. You cannot wrap a piece of paper into the shape of a torus without stretching the paper. We've tried wooden rings, paper circles, slinky toys, and computer graphics, but all of these only give a framework on which to drape a mental surface (**Figure 2**). But these are *not* actual toruses. Perhaps we should follow the mean advice of Eratosthenes: "Do not seek to do the difficult business of the cylinders of Archytas . . . ."

We recently were inspired by the fight

FIGURE 2



Computer graphic representation of torus.

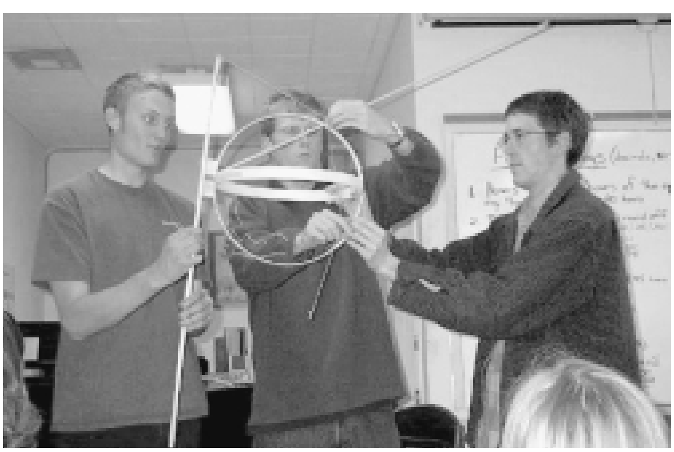
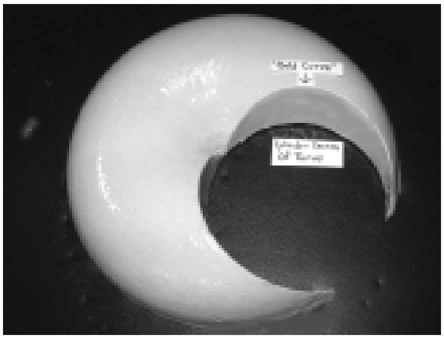


FIGURE 3

The torus-building machine tool, and its product.

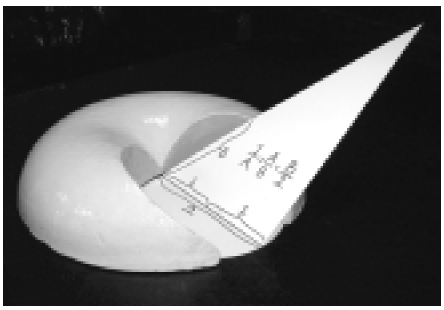
to save the automotive sector in the U.S.A., and we built a machine tool incorporating two layers of circular action, which carves out a toroidal bowl from some drying plaster of Paris. The tool we designed has a stack of compact discs (CDs) secured to the end of a long 3/8-inch bolt, with three CDs glued perpendicularly inside cuts at equal divisions

FIGURE 4



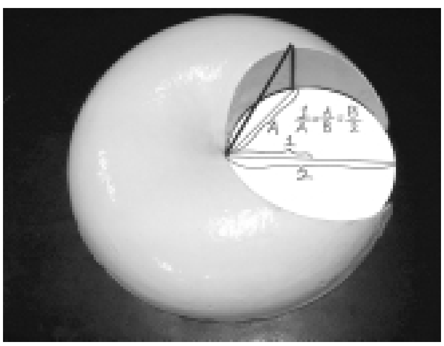
The bold curve, as an intersection between torus and cylinder.

FIGURE 5



The intersection of the cone and the bold curve produces the larger of two geometric means between 1 and 2.

FIGURE 6



The projection down of the intersection between the cone and bold curve, produces the smaller of two means between 1 and 2.

of the CD stack. To these CDs, we glued cardboard semicircles of the desired radius for our torus.

Then, as a large bowl of plaster is drying, we used a hand drill to sweep out a half-torus (**Figure 3**). We could then use this as a mold, to create positive toruses, one of which we produced with a cylindrical section cut from it.

The intersection of the actions producing the torus and cylinder, gives us a special curve, extending from the center of the torus to a point opposite the center, which Eudemus called the bold curve (**Figure 4**).

Now, sweeping out a particular conic action intersects this bold curve at a point which, when connected by a line to the center of the torus, results in a length equal to the larger of the two desired geometric means (**Figure 5**).

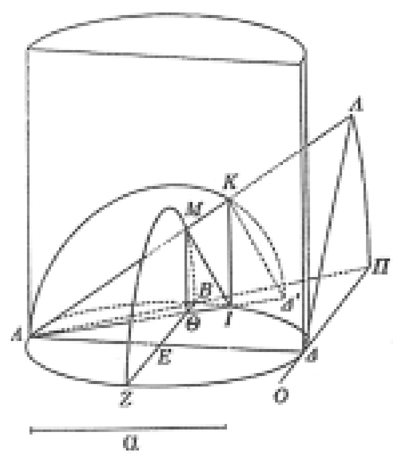
Projecting that intersection directly downward, to a plane that slices the torus in half (like a bagel), one obtains a second point, which, when connected by a line to the center of the torus, results in the smaller mean (**Figure 6**). If the two extremes, the radius and diameter of the cylinder's base, are 1 and 2, respectively, the shorter and longer means will give you the edge lengths of the doubled and quadrupled cubes, respectively.

Now look back at the problem. We wanted the means to build a doubled cube, and we ended up with a construction, using surfaces of revolution, to find a set of straight lines (**Figure 7**).

Isn't this strange? The *volumes* contained by the surfaces depend on a different principle than the volume of the cube. Nevertheless, it is the intersection of these *surfaces* that gives us means to double the cube. We're using two lower orders of magnitude, to produce a higher-order magnitude. This is like using the right combination of pork chops to construct a New York strip, or finding the right combination of dolphins and chimpanzees that produces a human being. Yet, here we are using lines and surfaces, to build a volume! This is not only strange, but, *paradoxical.*

Let's think like Archytas—who developed his ideas of mean proportionals from investigating music—and

FIGURE 7



Eudemus's drawing of the Archytas construction. The intersections of the surfaces give straight lines, not volumes!

invert the construction. Perhaps the arrangement of the three circular actions, is determined top-down, rather than bottom-up. In this case, the intersection point is not caused by an adding up of three surfaces, just like a musical interval is not note plus note. Instead, Archytas arranged them to reflect a process that is not continuous in the visible domain. Imagine a cube, growing continuously into a cube eight times the volume, passing through the doubled volume. Archytas' arrangement of actions thus captures two snapshots, the doubled and quadrupled cube, and pulls them from the invisible continuous process, into the visible domain.

The torus, cylinder, and cone are footprints of this act of making the invisible, visible. So is the sphere, which is also a surface generated by two orthogonal circular actions. Thus, the construction of the two means between any two extremes can be represented on the sphere. But, the sphere does not have the ability to generate those means by itself. The construction of the means requires the *unfolding* of the spherical action, *by Man*. Metaphorically, Archytas' discovery, and our little machine tool, formed the two means between the invisible domain of continuous cubic growth, and the visible domain.

—*Peter Martinson*