er in 1676, a branch of science which, together with the mastery of the implications of elliptical functions, had previously been assigned to future mathematicians by Kepler. The roots of Kepler's prescription had been the implications of the method which he had proven conclusively by the characteristic, internal features of his own absolute originality in his discovery of universal gravitation. (See **Box 10.**)

The general, relatively widespread knowledge of Kepler's discovery of universal gravitation among readers in England, had been made available prior to the misleading bowdleriza-

Box 10

Kepler's Approach

"Anyone who shows me my error and points the way will be for me the great Apollonius."

—Johannes Kepler, Astronomia Nova

Kepler's anti-Euclidean approach to astrophysics dealt not with the motions of the heavenly bodies, but with the power that caused their motion. Shapes, figures, forms, and curves—none of these were adequate to express a principle that caused motion. Kepler dispenses with the empiricist approach of Ptolemy, Copernicus, and Brahe in the first section of his *Astronomia Nova,* demonstrating that while their three systems appear to differ, they are all geometrically equivalent, and therefore, all wrong. For how can figure cause itself?

Kepler's adoption of *metaphor,* in his revival of the Greek approach of *Sphaerics,* called for something that is not a shape, curve, figure, or any other geometric object expressed in sense-perceptual terms: *gravitation.* In developing his hypothesis of universal gravitation and his working-through of the operation of this idea ("species"), he lawfully pushed the inadequate geometric language of his time past its limits to the point of collapse:

Kepler hypothesized that planets move in ellipses at a speed inversely proportional to their distance from the Sun due to the weakened power of gravitation at greater distances (**Figure 1**).

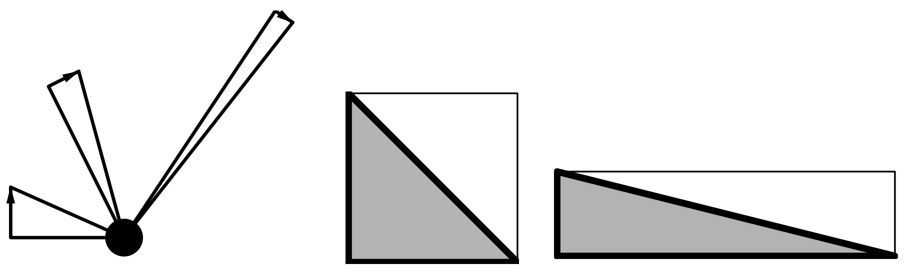
A problem arises in implementing this idea: Since a planet's direction changes at every moment, how small must these triangles be, and how many are needed to be a perfectly accurate measure of time? If the triangle has any size at all, does it not presuppose linear action in the small, and eliminate constant change? Kepler transforms the idea of an infinite number of triangles of motion, each seemingly so small as to be "nothing," into a continuous area swept out between the planet and the Sun, which idea Kepler uses as a measure of time (**Figure 2**).

Here, planet *P* has moved a distance of arc *A* from point *O,* sweeping out an area *SPO,* which area is a measure for the time of the motion. This area consists of both a circular sector *CPO* and a triangle *SCP.* While the area of circular section *CPO* is measured by the length of arc *A,* the area of triangle *SCP* is measured by *h,* the *sine* of arc *A.*

As Cusa had demonstrated over a century earlier, these two magnitudes, *A* and *h,* are incommensurable. Given a position *P,* it is possible to measure and determine the enclosed area, but, given a

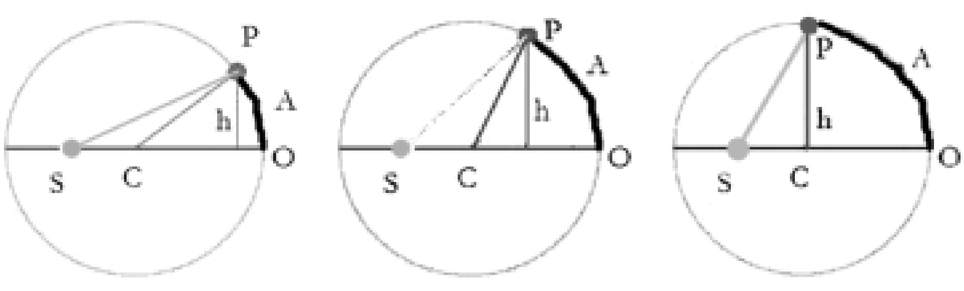
Box 10 continues on next page

FIGURE 1



The distance a planet moves in a period of time is inversely proportional to its distance from the Sun. The same given interval of time results in triangles of equal area. For example, at a distance (radius) twice as far from the Sun, the motion per time interval (arrowed change) is only half as far. This makes a triangle of double length but only half the height, which is therefore the same area. This area is a measure for time.

FIGURE 2



tion of Kepler's work by, ostensibly, Isaac Newton. To the extent of the relevant biographical evidence available, to the end of his life, Newton had no relevant knowledge of what a calculus is to the end of his life.

To situate the subject of the implied attacks, by D'Alembert, Euler, Lagrange, et al., against the physical relevance of Archytas' solution not only for the Delian paradox, but that paradox's relevance for all competent modern science and statecraft,

desired area, is it possible to exactly determine *P?* Kepler found this task of determining the *exact* position of a planet at a future time to be impossible:

"And while the former [circular sector] is numbered by the arc of the eccentric, the latter [triangle] is numbered by the sine of that arc. . . . And the ratios between the arcs and their sines are infinite in number. So, when we begin with the sum of the two [the sought area as a measure for time], we cannot say how great the arc is, and how great its sine, corresponding to this sum. . . . I exhort the geometers to solve me this problem: 'Given the area of a part of a semicircle and a point on the diameter, to find the arc and the angle at that point, the sides of which angle and which arc, enclose the given area.'. . . It is enough for me to believe that I could not solve this a priori, owing to the heterogeneity of the arc to the sine. Anyone who shows me my error and points the way will be for me the great Apollonius."

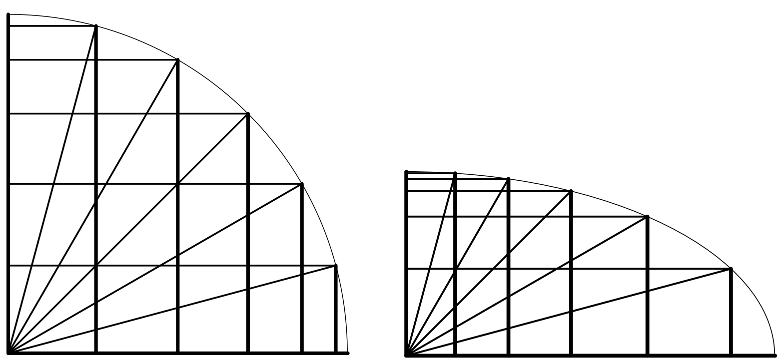
The "error" lies not with Kepler, but with the underdeveloped language he was using. He had developed a physical principle that lay between the "cracks" of geometry, but his mathematical language was one of figures, not principles. The cracks between his triangles were mathematical anomalies, but reflected an ever-present physical cause. It remained for Leibniz to introduce *metaphor* (*dynamics*) to create a physical mathematics adequate to address *physical,* rather than merely *mathematical* questions.

Kepler's challenge to the future prompted Leibniz's mastering of "nothings," such as the cracks between Kepler's area triangles, in his uniquely original discovery of a truly infinitesimal calculus (**Figure 3**).

Leibniz's calculus was not enough. The double incommensurability of the ellipse defied Leibniz's attempts at expression by circular functions. A fuller understanding of the higher classes of elliptical and hyper-elliptical transcendental functions would have to await the work of Gauss, Abel, and Riemann, over two centuries after Kepler.

—*Jason Ross*

FIGURE 3



Circular and elliptical quadrants. The length of arc along a circle is directly measured by the angle of rotation from the center, while the lengths of the sines (vertical lines) change unmeasurably. On the ellipse, the incommensurability of the sine continues to exist, as well as another: The length of arc is no longer measurable by the angle of (circular) rotation at the center. (Is it fair to even consider rotation on an ellipse from the standpoint of constant circular *rotation?) Can a magnitude be* doubly *incommensurable? If so, what is creating it, for how could an already understood principle create something incomprehensible?*