The Shadow of 'Power'

Look at the way in which silly reductionists, such as de Moivre, D'Alembert, et al., reacted to the encounter with what they called "imaginary" roots appearing within those cubic functions on which D'Alembert et al., focussed their attack on Leibniz's discovery of the catenary-linked universal principle of universal least-action, the fundamental physical principle of the Leibniz calculus as a whole. (See **Box 13.**)

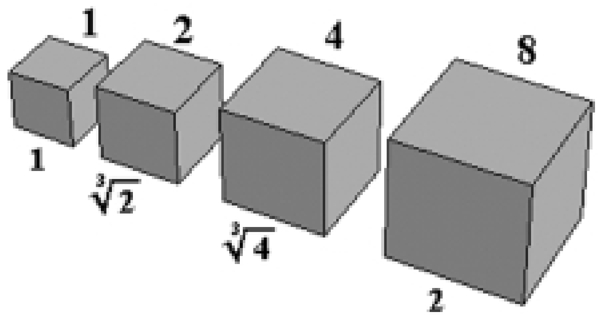
Now, consider the opening several elements of the expression of a "Fundamental Theorem of Algebra" in Gauss's 1799 doctoral dissertation. Compare this series of terms with the Pythagorean notion, defined in terms of *Sphaerics*, of the dis-

Box 13

How Cubic Roots Are Defined Algebraically

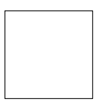
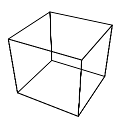
From the Greek studies of the line, square, and cube came an understanding of simply, doubly, and triply extended self-similar action. For example, the triply extended action of a cube necessitates two means between the extremes. This gives an idea of cubic roots (**Figure 1**).

FIGURE 1



It is easy enough for us to retrospectively apply the symbols x, x^2, x^3 to lines, squares, and cubes, respectively. But to what geometry do x^4, x^5, etc., correspond? (**Figure 2**)

FIGURE 2

x	x^2	x^3	x^4
			?

One solution to this paradox (preferred by petulantly childish formal mathematicians) is shown in **Figure 3**:

FIGURE 3

x	x^2	x^3	x^4

Ah, what a relief—with that pesky geometry out of the way, we can enjoy the unfettered freedom of manipulating symbols with assumed self-evident properties! We can simply recognize that x^3 means x times x times x; no troubles here! We can add and subtract too! $5-3 = 2$. And if we want $2-6$, we'd get -4. Hmm, that's a new type of number I did not mean to make with my self-evident numbers, but what of it?

Continuing, we can make equations: like $x^2 = 4$, which we can solve with $x = 2$, and also our "negative" number $x = -2$. We could even say $x^2+4 = 0$, which has as its answer. . . . Well, let's see. . . . Using the rules of algebra, $x^2 = -4$, but what on earth squared is -4? Both 2^2 and $(-2)^2$ are $+4$, not -4. Well, even if it makes no sense, we can use our rule to take the square root of both sides and get $x = \sqrt{-4}$. Now, this corresponds to no real magnitude, but, who cares? Let's use it anyway!

In fact, looking at $x^3 = 8$, we get no less than three solutions, only one of which even makes sense: 2, $-1+\sqrt{-3}$, and $-1-\sqrt{-3}$! Where are these strange numbers coming from? What is the source of these foreign intrusions into *my* view of the universe? Don't I have the personal right to look at things from my own point of view?

—*Jason Ross*