ence seeks to perfect a mathematics reflecting the distinct species of physical composition in the universe as a whole. Exploring the elementary distinctions among point, line, surface, and solid is the anteroom of physical-scientific thinking as a whole. In this aspect of the subject, the nastiest of all problems has been the conception of the *point.* What, physically, is

Box 16

Eratosthenes' Sieve

"First of all, though they had eyes to see, they saw to no avail; they had ears, but they did not understand; but, just as shapes in dreams, throughout their length of days, without purpose they wrought all things in confusion. They had neither knowledge of houses built of bricks and turned to face the sun nor yet of work in wood; but dwelt beneath the ground like swarming ants, in sunless caves. They had no sign either of winter or of flowery spring or of fruitful summer, on which they could depend but managed everything without judgment, until I taught them to discern the risings of the stars and their settings, which are difficult to distinguish.

Yes, and numbers, too, chiefest of sciences, I invented for them, and the combining of letters, creative mother of the Muses' arts with which to hold all things in memory. . . ."

—Prometheus, speaking in Aeschylus's Prometheus Bound

This astronomical origin of number and its connection to man's economic development, enunciated by Prometheus, is at the heart of the only truthful approach to science. Nevertheless, since that time, Zeus's would-be minions, who have sought to prevent the emergence of new Prometheans, have tormented countless generations by substituting for this physical-geometric origin of number, a sophistical form of arithmetic that associates number with merely the counting of things. Thus, the restoration of sanity in economics, so urgently needed today, is linked to jettisoning those infantile notions of arithmetic, used by bankers, accountants, and statistical physicists, replacing such foolishness with the higher notions of number associated with Plato, Eratosthenes, Cusa, Fermat, Leibniz, Gauss, Dirichlet, and Riemann.

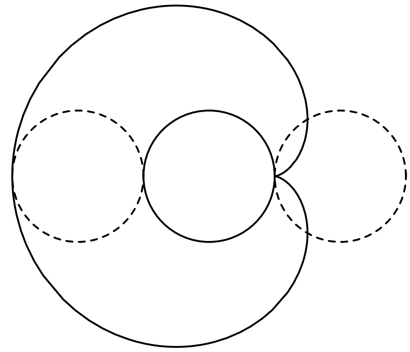
A simple pedagogical way to begin to demystify number's astronomical origins, and restore mental health to the victims of digital computers, is to examine the example of the most recognizable astronomical cycles, the Earth day, lunar month, and solar year. Each cycle is a physically completed action. Thus, each cycle lays claim to the number one. Yet all three exist in One universe. As such, there must be a greater One that subsumes these *relative* ones. Number, as Plato, Eratosthenes, Cusa, Leibniz, and Gauss understood it, unfolds from such relationships among these *relative* ones when they are considered with respect to a greater unity. This is why Cusa said, in *On Conjectures,* "The essence of number is the prime exemplar of the mind."

Thus, when one of these cycles is considered as one, the others become multiples of that one. For example, when the Earth day is taken as one, the lunar month contains a multiple of days. After 29 Earth days the lunar cycle is almost complete, but not quite. The Earth will complete another cycle before the Moon completes its cycle. From this standpoint, one lunar month and one Earth day are *relatively* incommensurable. However, after two lunar cycles, the Earth and Moon will return to their original orientation.

Now add the solar cycle. Compare that with the lunar and Earth cycle individually, and all three together. Note the mutual commensurability and incommensurability of the cycles.

From this type of astronomical-physical determination of number, the Pythagoreans understood the existence of two species of numbers: the rational numbers associated with cycles that ultimately become commensurable, and irrational numbers associated with cycles that are inherently incommensurable.

FIGURE 1



To grasp this point, think of two cycles, represented by circles of equal sizes. Allow one circle to roll along the circumference of the other. After one rotation of the rolling circle, the two circles will be in the same relationship as at the beginning of the cycle (**Figure 1**). Now, let the diameter of the rolling circle decrease, and examine the effect of this decrease on the commensurability or incommensurability of the cycles. There will be some relationships in which the two circles are incommensurable (**Figure 2**). There will be others in which the rolling circle completes its cycle after a finite number of rotations. These commensurable numbers are called whole numbers, 1, 2, 3, 4, . . . and rational numbers, 2/3, 5/4, etc. (**Figure 3**).

But this is a "bottom up" approach. Now look at the same generation of numbers from the "top down." Instead of creating these rational proportions by first creating whole numbers 1, 1+1, 1+1+1, etc., begin with a concept of the One and derive the whole numbers as parts. To express this geometrically, take a circle as the One and divide it. Halving the circle produces two parts, and thus the number 2. Halving again produces four parts, and the number 4, halving again eight parts, etc. But while this process will produce a *point?* That, Euler seems never to have understood, which is why he joined the reductionist horde in his savage, and also intellectually childish attack of 1761 on Leibniz. (See **Box 17.**)

Actually, a point is a kind of idea corresponding to an image of an anything which attempts to appear to be nothing.

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FIGURE 2

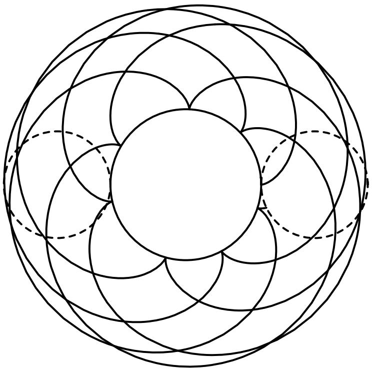
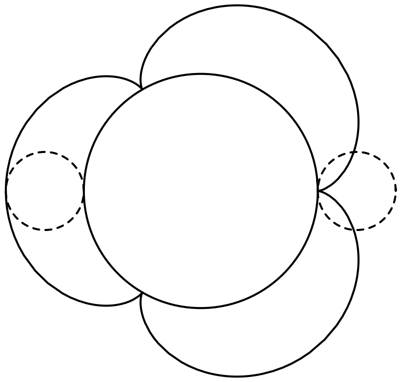


FIGURE 3



ever greater divisions of the circle, and the series of whole numbers, 2, 4, 8, 16, etc., such a process will never divide the circle (One) into three parts.

To divide the circle into three parts, and thus obtain a concept of the number 3, requires an entirely different action. Once this is accomplished, the three parts can be halved to produce 6 parts, and halved again to produce 12. Also, each of the three parts can be divided again into three parts producing 9, and continuing to 27, etc. From this process the divisions into powers of 3, powers of 2, and multiples of the powers of 3 and 2 are formed. But such a process, although producing an infinitude of possible divisions, will never divide the circle into 5, 7, or 11 parts.

These types of numbers, 2, 3, 5, 7, 11, etc., which cannot be formed by combinations of other divisions, but from which other divisions can be formed, were recognized by the Greeks as the "prime" numbers. Thus, the prime numbers are the numbers from which all other numbers are made.

The very existence of prime numbers is already an indication of the foolishness of thinking of numbers generated by the childish method of adding 1, and defining an arithmetic by the formal operations of addition, subtraction, multiplication, and division. Each such operation, rather than being a set of rules, must be understood, as the very existence of prime numbers attests, as a different type of physical action.

A still deeper concept is revealed when one seeks to find the cycle that produces prime numbers. From the bottom up approach of adding 1, the prime numbers seem to appear suddenly without warning. Sometimes two appear near each other, such as 11 and 13, and sometimes there are several numbers in between, such as 23 and 29. While the density of the prime numbers decreases as the numbers get larger, they never cease to appear.

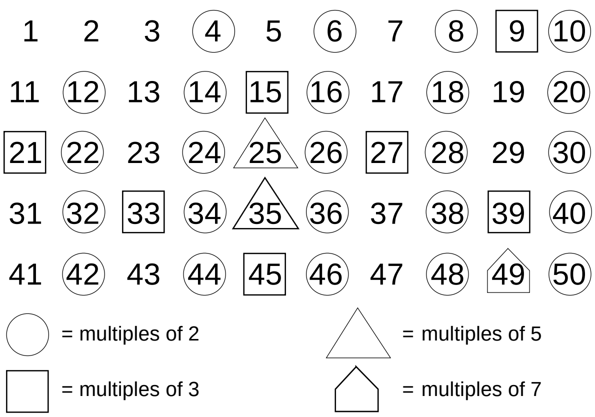
Thus, to even find the prime numbers—*the numbers from which all other numbers are made*—the bottom-up approach must be abandoned for the domain which Gauss called "higher arithmetic." That domain treats the entire class of numbers as a One, and all numbers are considered with regard to their relationship to that One. But since the number of numbers is infinite, we must think of that One, from the physical-geometric conception of number associated with the astronomical origin of number enunciated by Prometheus.

A Higher Concept of Number

This higher concept of number is expressed by the method of finding the prime numbers created by Eratosthenes, which he called a "sieve." The sieve takes all the numbers as its beginning, and extracts the primes in a similar manner to the above illustration of the divisions of the circle.

To construct Eratosthenes' sieve, create an array of numbers from 1 to any upper bound. Then, beginning with 2, pull out from the array all multiples of 2. Then go to the next highest number that was not extracted, which would be 3. Extract from the array all the multiples of 3. When this is exhausted, go the next highest number after 3 that was not extracted, which would be 5. Continue this process. The sieve will extract all prime numbers from the array (**Figure 4**).

FIGURE 4



In this way, the existence of a more complex cycle begins to emerge, the cycle of prime numbers, that reflects the complex geometrical structure of the physical universe itself. That structure was investigated further by Fermat, Gauss, Dirichlet, and Riemann. The depth of those insights is beyond the scope of this short report, but their investigation, as Plato said, draws the mind closer to truth and being.

—*Bruce Director*