Box 17

Euler Misses the Point

"The monad . . . is nothing else than a simple substance, which goes to make up composites; by simple, we mean without parts. Now, where there are no constituent parts, there is possible neither extension, nor form, nor divisibility. These monads are the true atoms of nature, and, in fact, the elements of things."

—Gottfried Leibniz, The Monadology

In a direct attack on this concept of the monad and its author, Gottfried Wilhelm Leibniz, Leonard Euler wrote, in a 1756 letter to a German Princess, an argument to disqualify those who "insist that division extends only to a certain point, and that you may come at length to particles so minute that, having no magnitude, they are no longer divisible. These ultimate particles, which enter into the composition of bodies, denominate *simple beings* and *monads.*"

"This property [of division] is undoubtedly founded on extension; and it is only insofar as bodies are extended that they are divisible and capable of being reduced to parts."

"You will recollect, that in geometry it is always possible to divide a line, however small, into any number of equal parts."[1]

"Whoever is disposed to deny this property of extension is under the necessity of maintaining that it is possible to arrive at last at parts so minute as to be unsusceptible of any further division, because they cease to have any extension. Nevertheless, all these particles taken together must reproduce the whole, by the division which you acquired them; and as the quantity of which would be *nothing,* a combination of nothings would produce quantity, which is manifestly absurd! For you know perfectly well that in arithmetic two or more nothings joined never produce any thing.

"This opinion, that in the division of extension or of any quantity whatever, we come at last to particles so minute as to be no longer divisible because they are so small or because quantity no longer exists, is therefore a position absolutely untenable."

But wait a minute! This argument by Euler against the monad sounds suspiciously like a familiar argument made by Gottfried Leibniz in his *Dialogue on Continuity and Motion* years before, where he poses this problem:

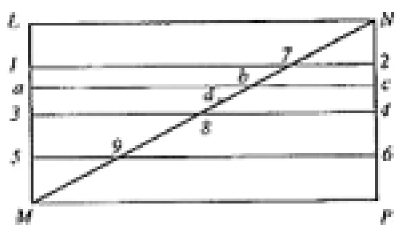
Pacidius: In a rectangular parallelogram, let a diagonal *NM* be drawn (**Figure 1**). Isn't the number of points in *LM* the same as the number in *NP?*

Charinus: Without doubt. For, since *NL* and *MP* are parallel, *LM* and *NP* are equal.

Pacidius: Now, any horizontal line drawn from a point on the line *LM* to the line *NP* will have a corresponding point on NP as well as on the diagonal *NM.* However, either there are extra points on the diagonal *NM* which could not be intersected, or the line *NM* has the same number of points as *LM* and *NP,* which would be absurd! However, conversely, one can draw a horizontal from any point left on the diagonal to a corresponding point on each of the sides! Whence it is established that lines are not composed of points.

So wait, what's going on here? Leibniz, the author of *The Monadology,* the paper which first laid out not only the existence, but also several of the main characteristics of monads extensively, argued for infinite divisibility and the impossibility of lines made up of points! So, both the subject of Euler's attack, as well as the attack itself came from Leibniz! Now, ask yourself this: Could it be possible that an 11-year student of Jean Bernoulli just didn't realize this?

FIGURE 1



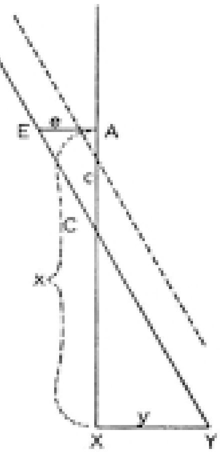
Maybe Euler, intentionally or unintentionally, missed the point.

Let's look at some other points:

Leibniz posed this investigation in a different way in a letter to Pierre Varignon in 1702, where he describes the following construction:

"Let two straight lines *AX* and *EY* meet at *C,* and from point *E* and *Y* drop *EA* and *YX* perpendicular to the straight line *AX.* Call *AC, c* and *AE, e; AX, x* and *XY, y.* Then since triangles *CAE* and *CXY* are similar, it follow that $(x-c)/y = c/e$ (**Figure 2**).

FIGURE 2



"Consequently, if the straight line *EY* more and more approaches the point *A,* always preserving the same angle at the variable point *C,* the straight lines *c* and *e* will obviously diminish steadily, yet the ratio of *c* to *e* will remain constant." (**Figure 3**)

What happens when *E* and *C* lie on *A?* (**Figure 4**)

At the vanishing point A, the relationships must still hold. But how can a point be a triangle? How many sides does this point have? Are all points created equal?

This type of true point can only be generated through a process, the denial of which is the real sophistry that Euler is employing. In a dead fantasy-mathematical world where points are just material nothings, you *can* divide anything *ad infinitum,* and free trade *is* good for humanity.

FIGURE 3

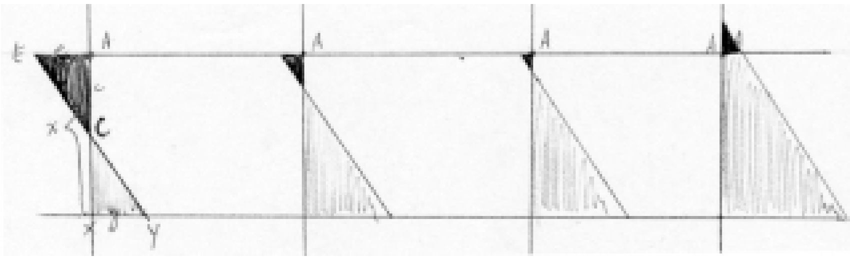
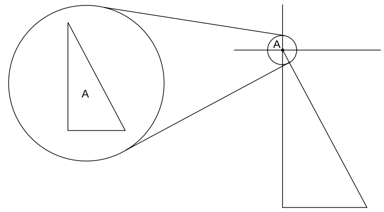


FIGURE 4



What is a point in the real world then?

Let's take a look at the problem of trying to divide the nation-state:

We begin with the nation-state itself, which was born as an expression of scientific breakthroughs in natural law, i.e., a body of people most closely organized according to the same principles as the universe itself, a self-governing, self-bounded entity. Now ask yourself how one could go about dividing the nation-state such that each part maintains the same sovereignty as the whole; or, as Leibniz put it, "because it [matter] is divided without end, every part into other parts, each one of which must have its own proper motion. Otherwise, it would be impossible for each portion of matter to express all the universe" (*The Monadology*).

The United States has 50 states, each with its own internal government, transportation system, power systems, agriculture, etc., and yet, each an integral part of the nation-state as a whole. The next such division is the county, and the city, with its own teachers, engineers, merchants, etc. Then we have the household, and finally, the individual citizen. The individual citizen is a sovereign entity, with the mind as its governing apparatus, and all its organs and arteries, which serve their own separate functions, but governed by a single intention, to serve the whole; an entire nation-state within one individual . . . or, is it the other way around? Has the nation-state been organized like the individual?! Such that the more diverse the occupations (organs), the more complex and efficient the operation of the whole; and each citizen, like the cells that make up all the parts of the body, are specialized but express one intention, the betterment of that whole.

To more clearly show the political attack by the mathematically imprisoned Euler, let's put him in power. How would he divide the nation-state?

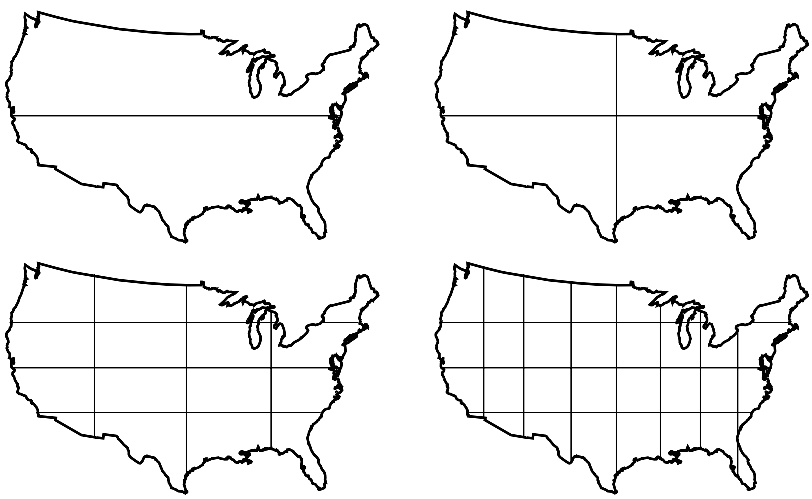
Here we go:

Divide the country into North and South sections. Then into Northeast, Northwest, Southwest, and Southeast, by drawing a line down the center vertically, then into eighths, sixteenths, and so on to infinity. (**Figure 5**)

Be careful not to get in the way, this may get bloody.

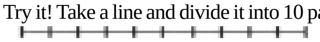
—*Liona Fan-Chiang*

FIGURE 5

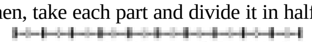


Notes

1. Try it! Take a line and divide it into 10 parts:

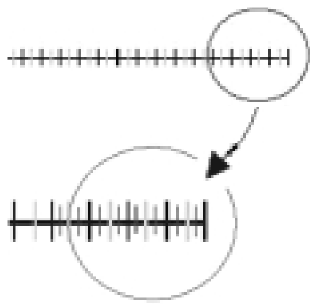


Then, take each part and divide it in half:



Now, these segments in turn can be divided in half again, and again, and again, into infinity, or until you get tired (you may need a laser).

In fact, no matter how small the segment gets, as long as it has any length, you could just get a magnifying glass and keep on dividing. "Hence it is affirmed that all extension is divisible to infinity; and this property is denominated divisibility in infinitum.



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